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ABSTRACT

This instructional guide and set of three companion workbooks are intended for use in an arithmetic course based on the Tic Tac Toe method of addition and multiplication, which is an alternative means of learning to add and multiply that was developed for students whose learning disabilities (including difficulty in distinguishing left from right and an inability to memorize multiplication tables) have prevented from learning arithmetic by traditional methods. Presented in the guide are the following: case studies illustrating the difficulties experienced by learning-disabled students when learning to add or multiply; basic concepts underlying the Tic Tac Toe method; the six patterns used in the Tic Tac Toe system; techniques for teaching and using Tic Tac Toe math and checking the patterns; and guidelines for using the grids to solve double-digit multiplication and division problems and producing double-, triple-, and multiple-digit multiplication tables. The first workbook contains exercises in distinguishing odd and even numbers, addition, the various positions of the Tic Tac Toe grids, and completing the grids and using them to solve addition, multiplication, and long division problems. The second and third workbooks contain exercises in double- and multiple-digit arithmetic and fractions. (MN)

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Tic Tac Toe Math

Instructional Guide

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Richard J. Cooper, Ph.D.

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This book is dedicated to Nanette, Jerry, Deron, Susie and Linda for whom I created this system. My thanks to them for allowing me to use it with them thus showing me how to teach it.

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INTRODUCTION

Many children and adults with learning differences, problems and disabilities have difficulty with math. Some of the reasons include: not thinking in quantitative categories, having difficulties with reversals, sequences and memory and simply not spending enough time on task.

Severe math problems are particularly evident with children and adults who still count on their fingers. They have never learned to add and also have had or continue to have difficulty remembering the multiplication tables.

People question why teach basic math facts when there are calculators? The answer is twofold. One reason is to build self-confidence in those who cannot perform as their peers do. The second reason is to provide the students with a better quantitative understanding of the world. Since much of our perception is based on categories that have been taught or conceptualized, students who do not have enough quantitative categories as part of their experience continue to be plagued with math problems.

The interplay of these problems often is seen in individuals with learning differences. For example, for students who have difficulty distinguishing right and left, memorizing facts is difficult because they have both the right answer (right) and the wrong answer (left) in their memory. Because of a progression of either/or options, they cannot remember which of the two answers is correct. Therefore, they guess sometimes correctly and other times incorrectly, increasing the progression. This problem is exacerbated under stress and both math tests and life situations involving math can be stressful.

The multiplication tables are particularly trouble some for those students who have the problems described above. Another reason many students do not learn the multiplication tables by memorization is insufficient time on task. They may spend time trying but wrong answers limit the value gained from their efforts. Additionally, avoidance of a difficult and sometimes painful (embarrassing) task reduces time on task.

These students need alternative means to learn to add and multiply, such as the innovative, effective system developed by Dr. Cooper, the Tic Tac Toe method.

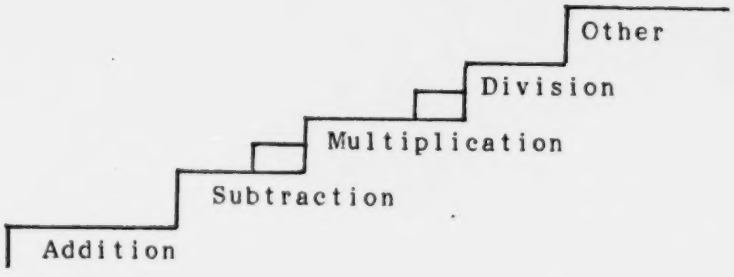
CASE STUDIES

Nanette avoided learning math by getting angry. She was a 12 year old girl who still had to count on her fingers to do even the simplest arithmetic problem. She tried without success to memorize the multiplication tables. The harder Nanette tried to learn her number facts the more frustrated she became because more often she got the wrong answer than the correct answer. After learning the Tic Tac Toe system for multiplication, her attitude about math and her self concept changed. She spent less time getting angry and more time on task. She became more open to math concepts and numbers in general. To her surprise, after one year using the Tic Tac Toe system, she has been able to remember many of the times tables without the grids.

Brad spend his summer trying to learn the times tables with flash cards, musical tapes and constant repetition without much success and a lot of frustration. After one hour of learning the Tic Tac Toe Math, he learned the times tables and proudly explained it to adults and his peers. His attitude immediately changed as if the weight of the world had been lifted from his shoulders.

BASIC CONCEPTS

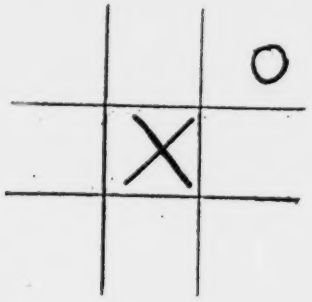
Some students need to memorize in smaller steps. The Tic Tac Toe method adds a step in the memory process. Just like a person does not jump to the top of a staircase, but rather gets there by going a step at a time, so also some students must have additional steps to get up to the next level. This system provides them with this step. It consists of memory clues which result in the student always (once the system is learned) getting the right answer. This lowers the frustration level and enables the student to complete higher level math with confidence. Once this confidence is achieved, the student can concentrate on other operations (e.g. word problems) which before were never dealt with because they were stuck on the multiplication tables.



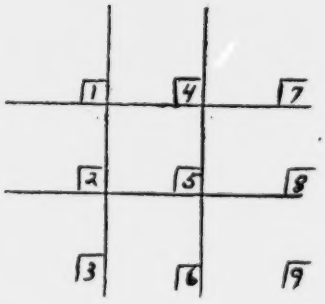
Many students have gaps or holes in the material they have learned which make the students stumble and fuels the frustration and avoidance. The following system helps them to relearn basic number facts because they will develop more confidence and thus be able to put more time on task.

(NOTE: Remember that years of avoidance and fear of failure are part of the past experience of students who have severe math problems. So a teacher must be very patient when teaching this system.)

Tic Tac Toe Math is based on patterns. These patterns are easily learned and they interrelate so that there are different ways of remembering them. Like the game of tic tac toe, this system utilizes the following grid.



The positions in the grid are numbered in the following way:



These positions are learned by constant repetition because they never change. The one position is always the number of the table and 1 times the number is a reminder that this is the place to start. The position numbers are counted down. Some students, who have directional problems, may, at first, have difficulty learning to count down for the positions. However, the consistency of always counting down for the positions is eventually learned.

The positions are also learned as the one times table. The one times table is important to learn and should not be skipped over as too simple because it is used to create more complicated grids. (These are explained on page 41.)

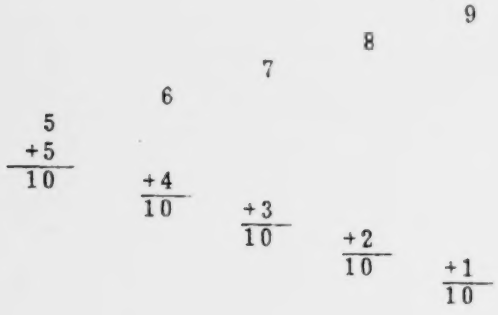
A couple of basic concepts are essential for the learning of this system. When teaching this system, always stress whether the number is odd or even. Many individuals who have severe math problems do not have a solid grasp of the odd and even numbers. This first should be checked. If the concept of odd and even numbers is weak, the teacher must teach and then reinforce this. If the student has a particular problem remembering these numbers, one way to teach the concept is to teach only the even numbers. Then the student is instructed that if a number is not one of the even numbers, it is odd. Additionally, the teacher must check to determine if the student understands which multiple digit numbers are odd or even. Some students will take the first number rather than the last to determine whether the number is odd or even. For example, the number 231 might be called even because of the two. The concept of odd and even will be reinforced by this system because the patterns are based on the difference between odd and even.

odd	even
1	0
3	2
5	4
7	6
9	8

Another concept which is important to know is the number facts which add to ten. Again students with severe math problems often do not know their number facts well. As a result, the numbers which add to ten need to be taught and reinforced. This system will also reinforce these basic number facts but the student may need a guide at the top of the page to facilitate the learning of the patterns.

5	6	7	8	9
+5	+4	+3	+2	+1
10	10	10	10	10

A good way to teach these number facts is through this visual pattern:



Reviewing the number facts that add to ten is a good beginning exercise for the student who is weak in this area. For those who do not have a problem with this area, they can just move on to the next step.

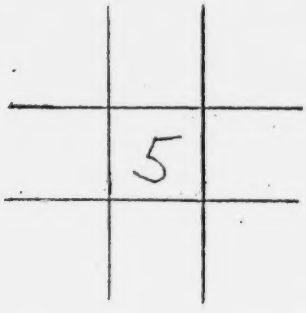
The number facts which are double the number are also important in this system and should be checked and reinforced. The student should notice (or be taught) that all these sums are even.

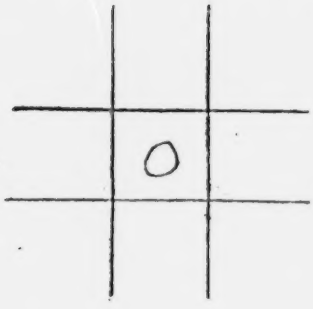
1	2	3	4	5	6	7	8	9
+1	+2	+3	+4	+5	+6	+7	+8	+9
2	4	6	8	10	12	14	16	18

PATTERNS

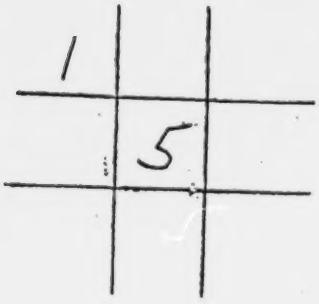
Pattern 1: The Center Position

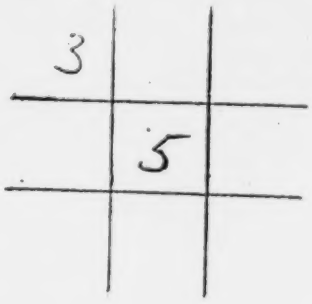
The first pattern that one notices in this system is in the center box. A way to impress the importance of the center box is to play tic tac toe with the student. This usually puts the student at ease and helps to establish a rapport. Even with adults, this techniques works. Explain that the center box is a key position in the tic tac toe game and it is also a key position the multiplication grid. The center position is always a <u>five</u> or a <u>zero</u>.

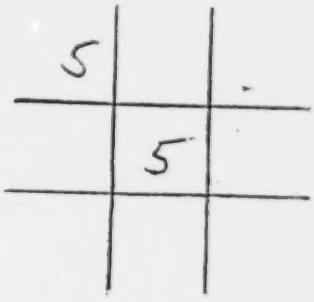


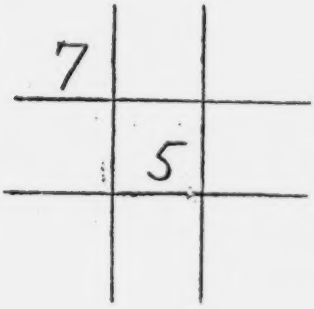


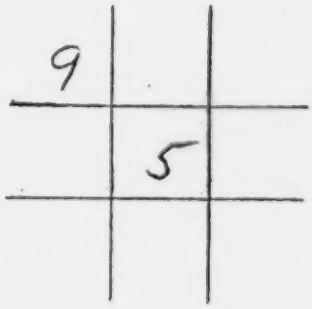
Five and zero appear in no other box in the 1, 2, 3, 4, 6, 7, 8, and 9 time tables. (Five and zero appear in other positions in the five and zero time tables.) The first pattern the student should see is in the center box. If the table is odd, there is a five in the center.



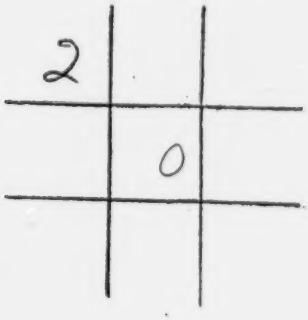


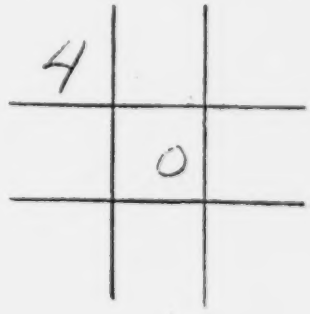


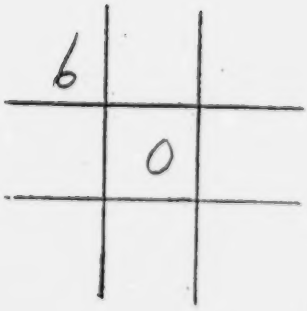


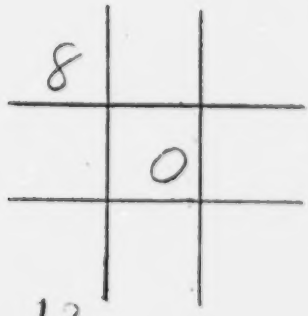


Zero is in the center position if the table is even.



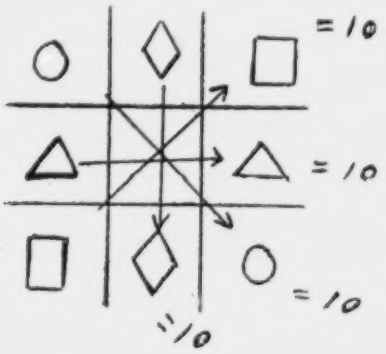


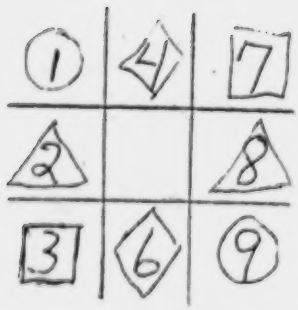


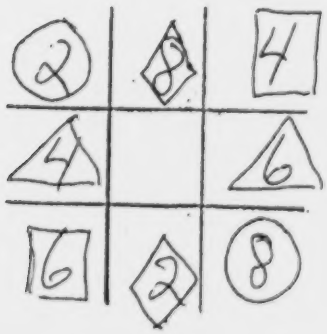


Pattern 2: Numbers adding to 10

Placing the time tables in the tic tac toe grid results in a number of patterns which are surprising. The second pattern which is noted is that the numbers in the one columns in the time tables (the numbers on the right of a two digit number) always add to ten in the following pattern:



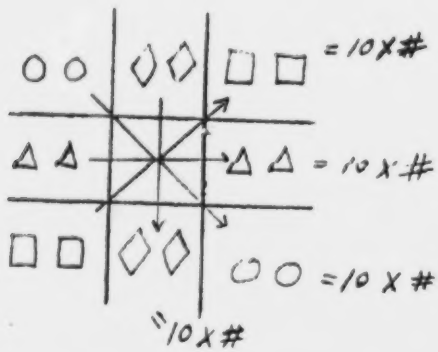


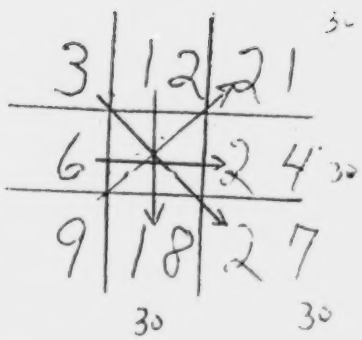


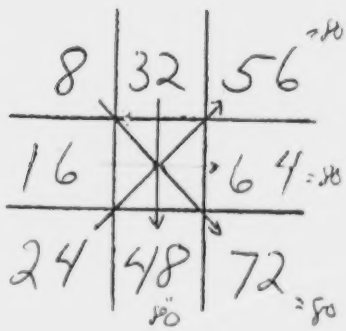
This is easy to learn for the student with a learning difference, problem or disability because it is consistent and simple and it works for all the tables. The more one uses the system, the more the number facts adding to ten are reinforced.

Pattern 3: Numbers adding to ten times the table

The next striking pattern is observed in the complete grid of all the time tables. The opposite positions (along the X and the +) add to ten times the first number in the table when the grid is completed.



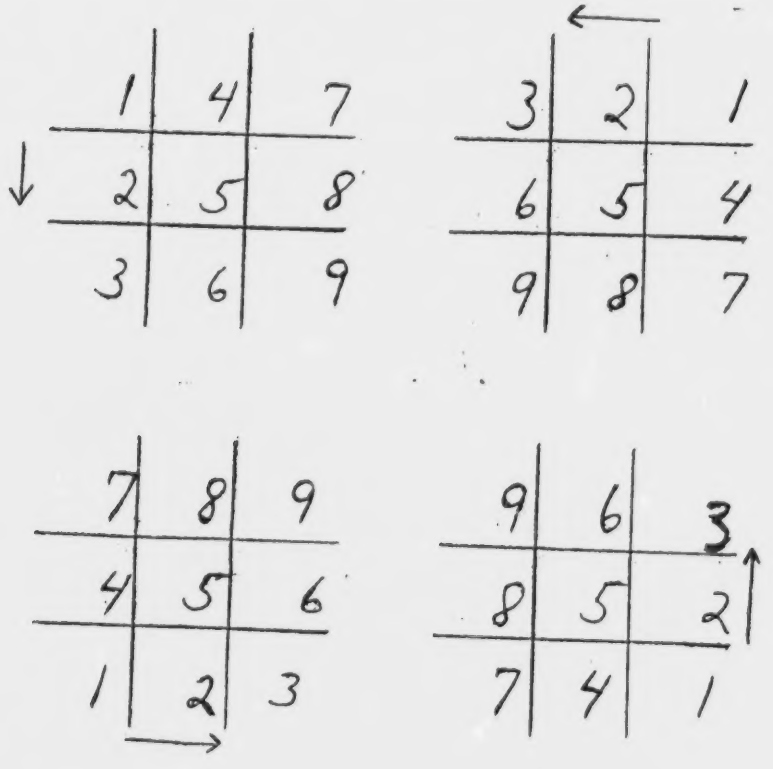




This pattern is not a necessary part of learning Tic Tac Toe Math but it provides the student with a way to check the patterns, especially when working with multiple digit times tables.

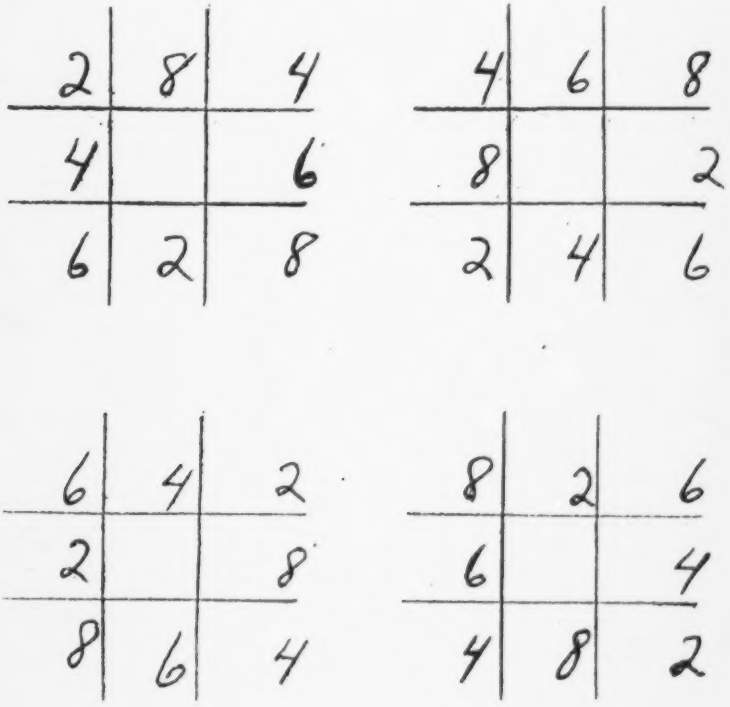
Pattern 4: The Order Of The Odd numbers

The next pattern which is noticed is in the tables of the odd numbers: 1, 3, 7 and 9. All the numbers, one through nine, appear in order. With each table, the direction of the order shifts a quarter turn. The number five is always in the center in each grid as was previously observed.



Pattern 5: The Order Of The Even Numbers

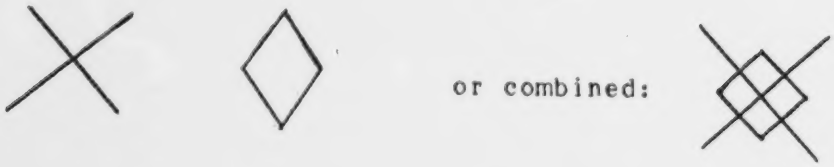
The pattern in the time tables of the even numbers is just as striking and also revolves a quarter turn. There are no odd numbers in the times tables of the even numbers and each even number (2, 4, 6 and 8) is repeated twice. There is always a zero in the center as has been previously observed.



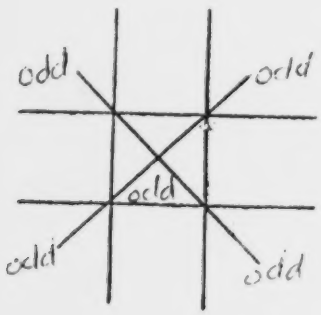
The rotation of the even numbers is not as easy to see and is not as important to know in learning the patterns. The rotation can be seen by looking at the corners.

Pattern 6: The X and the diamond

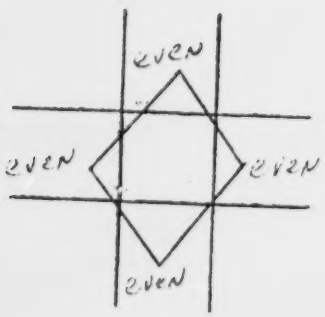
Another pattern that is observed in these grids is the X and diamond.



Along the X are the odd numbers

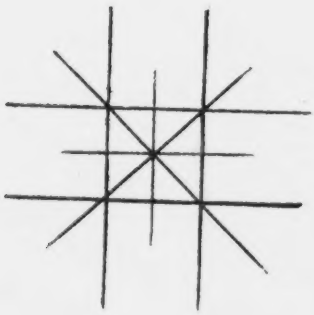


On the corners of the diamonds are even numbers.



These are visual patterns which for some students are easily learned. Another visual pattern is the cross and the X. This is the visual clue for the pattern which adds to ten.

or combined



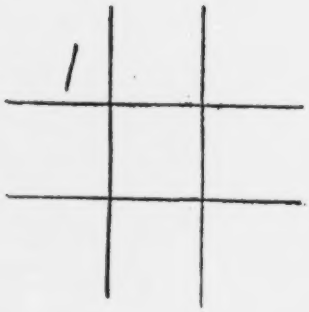
These patterns apply in all the tables for every number.

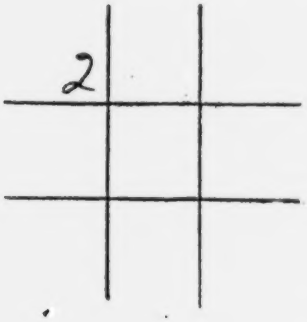
TEACHING THE TIC TAC TOE PATTERNS

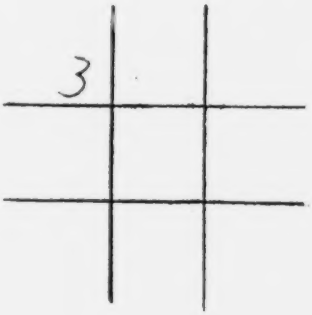
Those who do not have a severe learning disability, problem, or difference often cannot see the advantage to learning the patterns. If you are one such person, please refer to the introductory materials for an explanation of the rationale for this system. The patterns are more or less difficult to learn, depending on the nature of the person's learning difference and severity. Different methods or options are presented here. Any person using this book to teach another should utilize the option which makes the most sense to the student or the option which seems to be more easily remembered.

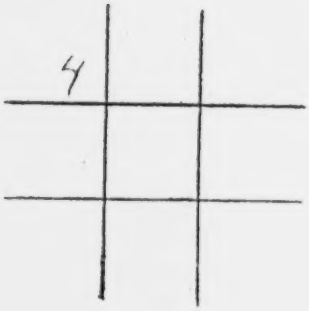
Some students can only learn one table at a time, while others can learn them all at one sitting. Adjust the speed of instruction to the individual needs of the students. It is often good to give examples of all the tables but only require the student to learn as many as he or she feels comfortable doing. Remember, years of avoidance may be adding to the problem in learning.

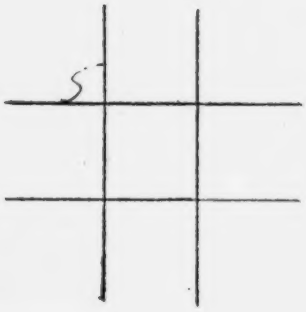
The first step in teaching the pattern is to have the student understand that the first position is always the number of the given times table, and that the one position is the same as 1 x that number.

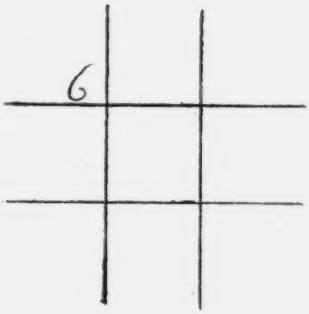


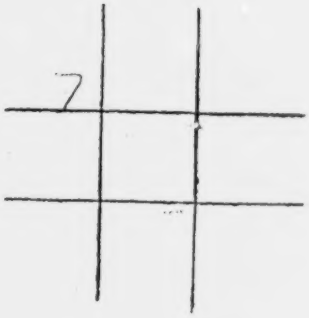


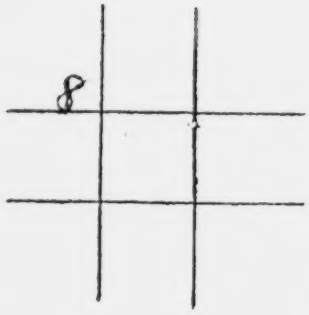


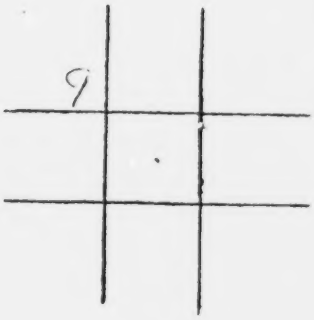












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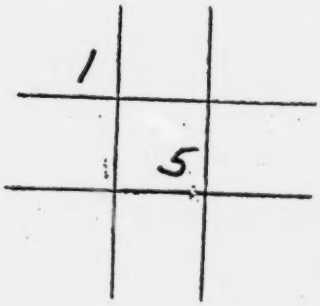
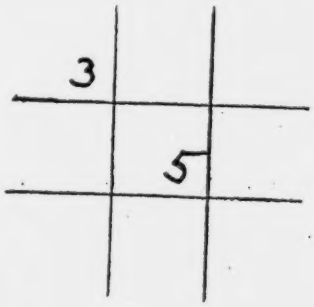
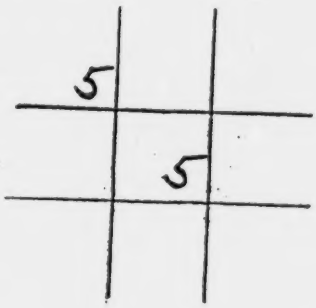
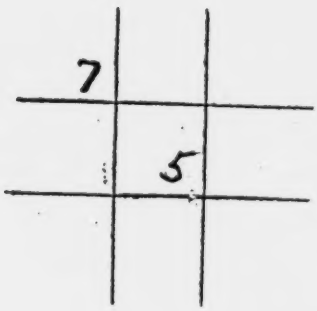
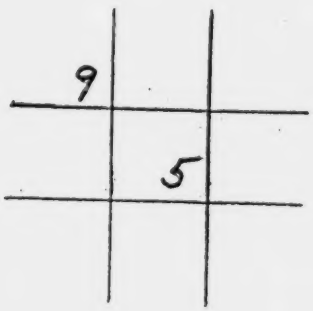
Second Step: The second step is to have the person decide if the number is odd or even. If it is odd, then a five is placed in the center position.



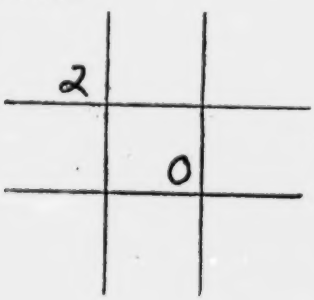


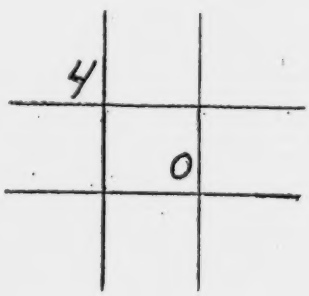


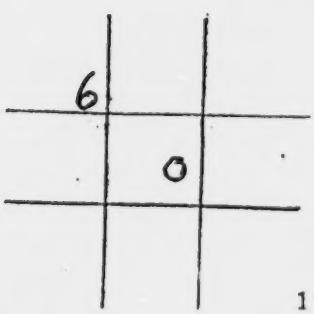


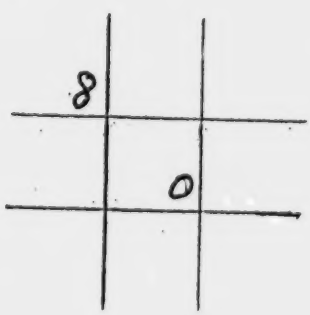


If the number is even, then a zero is placed in the center position.

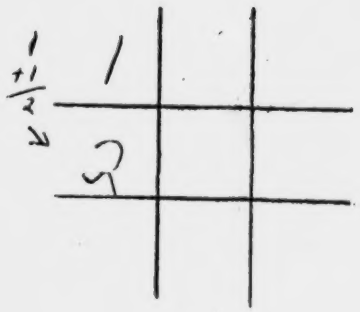


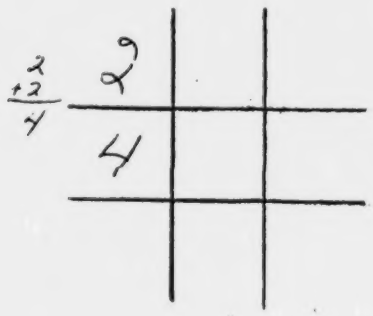


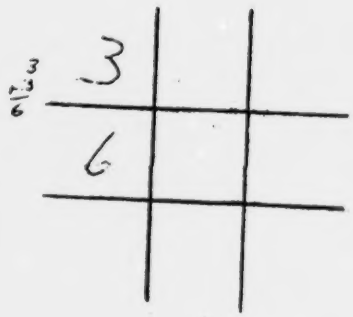


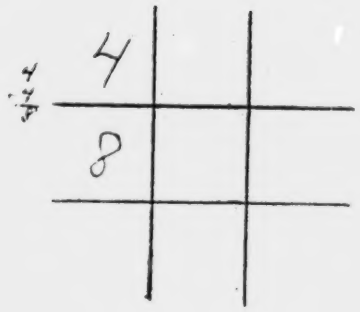


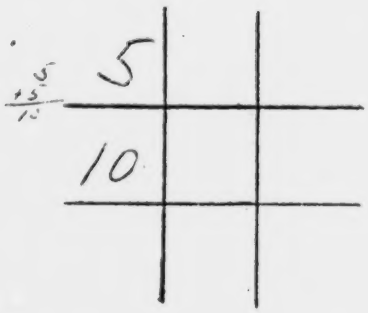
The third step in the teaching process is to show the student how to find the second number in the grid. This is done simply by doubling the first number. Most students can easily double the numbers up to 6. Some, with severe problems, often have problems doubling 7, 8 and 9. A memory clue for these students is that the sum of any number that is doubled is always even.

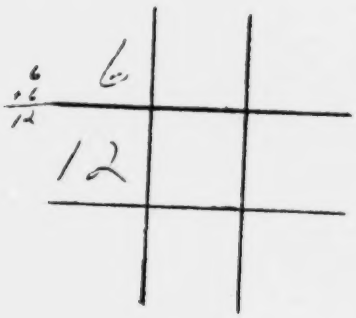


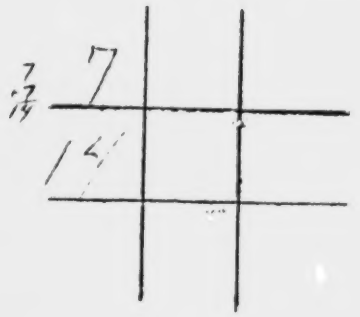


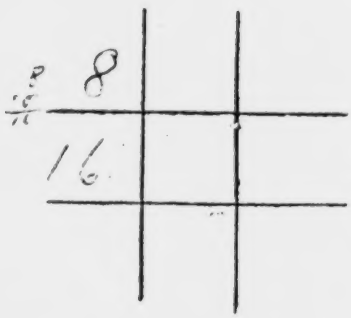


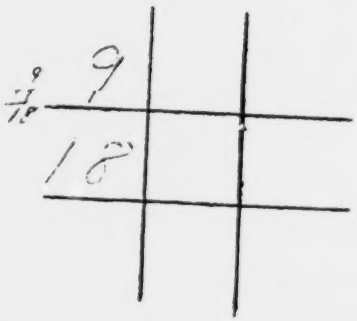












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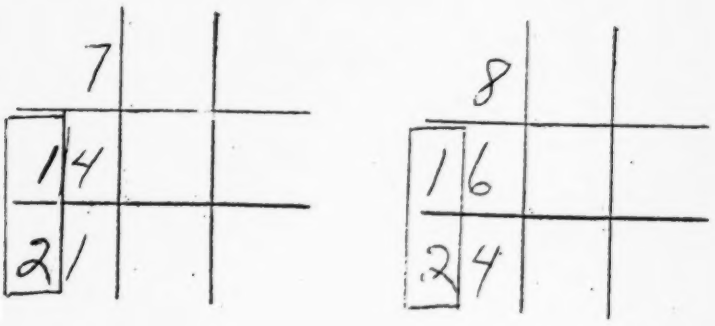
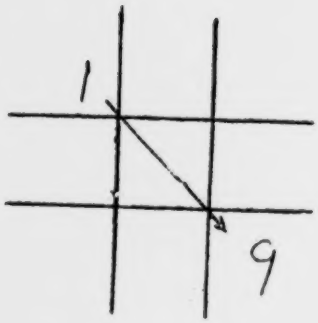
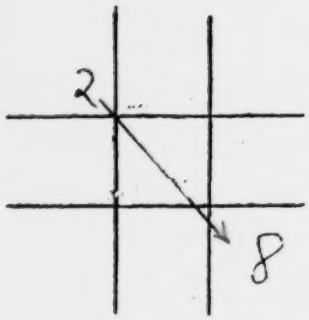
There are now two digits in some second and third positions. The student must be taught to ignore the number in the tens column (the number on the right) for the time being. The numbers in the ones column both add to 10 and follow the odd and even patterns.

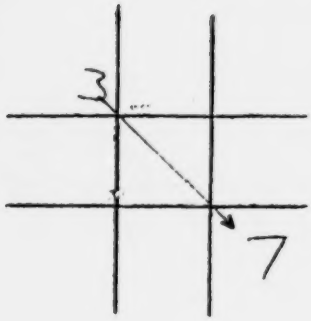


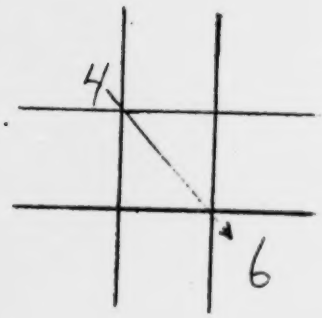
The digits in the ten's column will be added after all the single digits of the pattern are in place. Some students prefer not to write the ten column digits in the second or third place until the other positions have been completed. Others prefer to add these numbers whenever they are apparent. Either method is acceptable as long as the students does not become confused. (e.g. trying to find the number that adds to 14 to equal 10.)

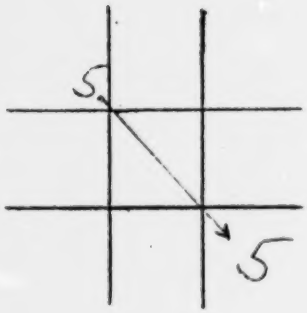
The fourth step is to find the diagonal to the first number. These numbers add to ten.

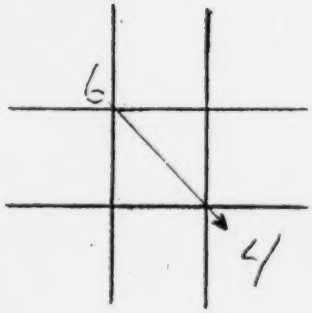


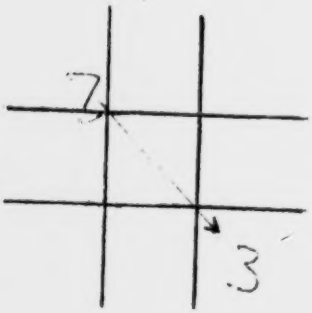


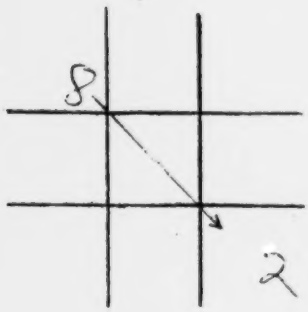


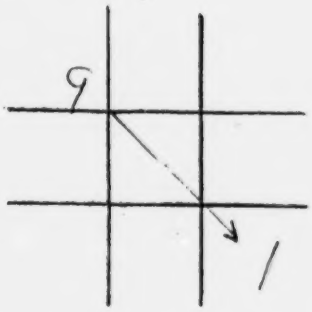












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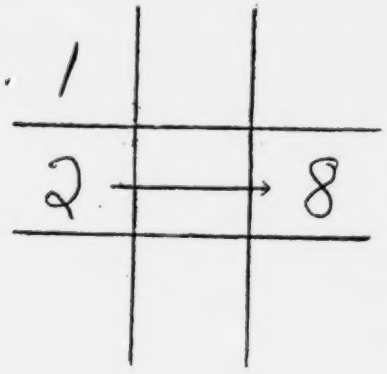
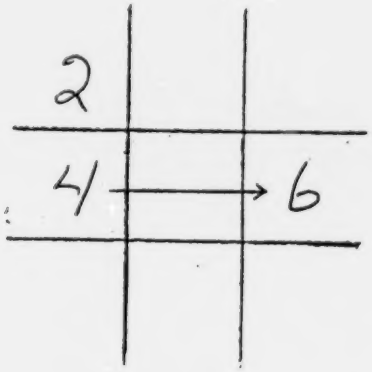
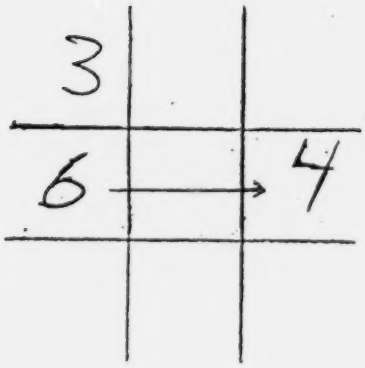
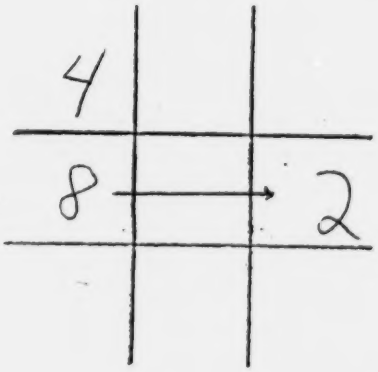
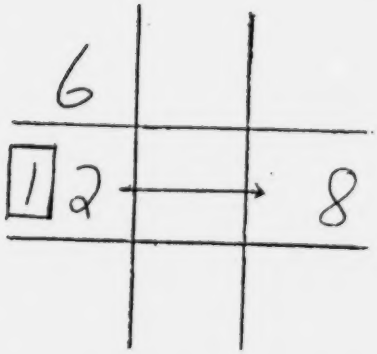
The number in the eighth position can now be added by using the pattern of adding to ten.

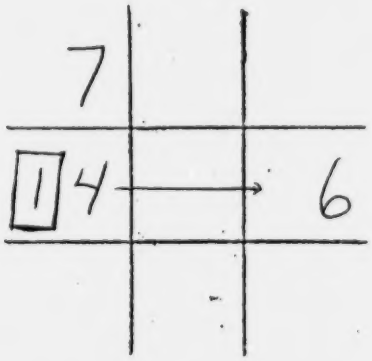


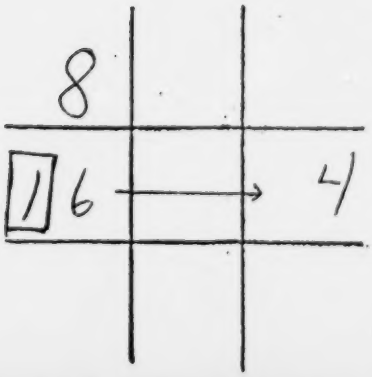


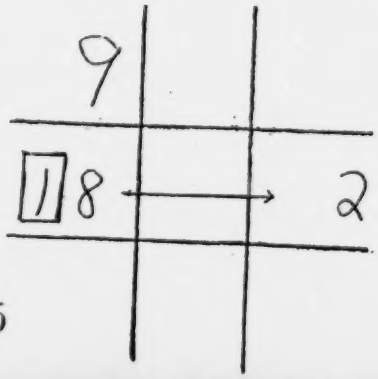






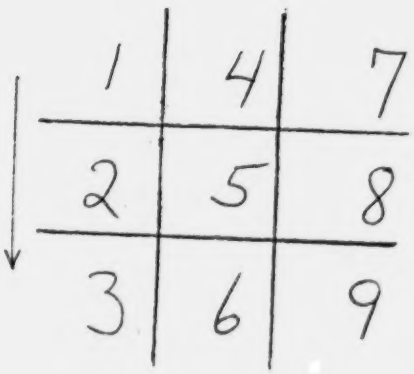


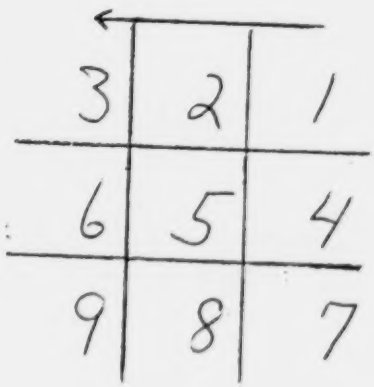


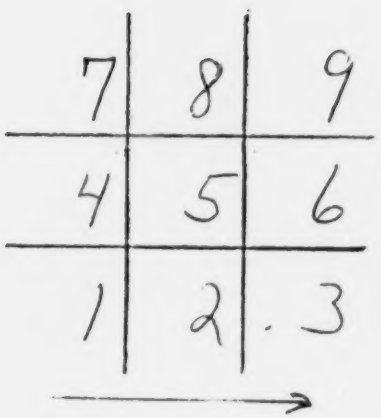


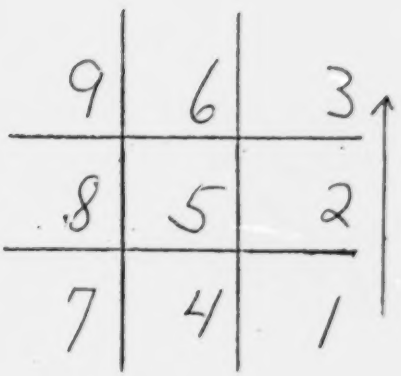
When students become more experienced, many have enough numbers to see the patterns in the grids and can complete them without further clues. These students now can see the pattern of the numbers being in order from one to nine in the odd patterns.

Occasionally the student will forget what number to place in the fourth and sixth positions. An arrow indicating which way the order is going will help them to remember what number goes where.

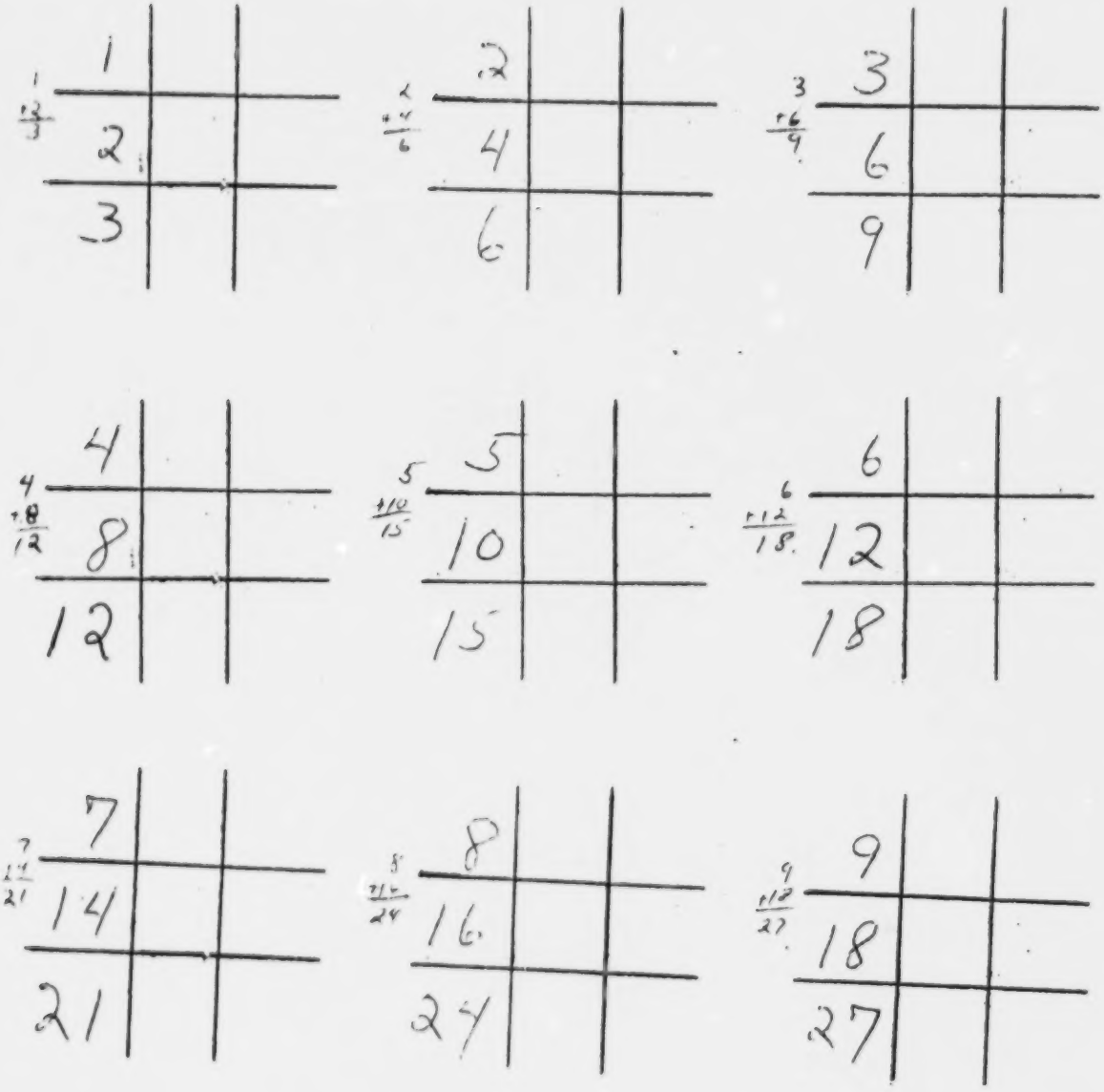




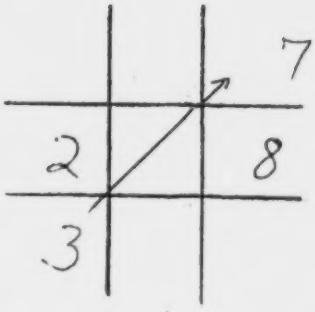


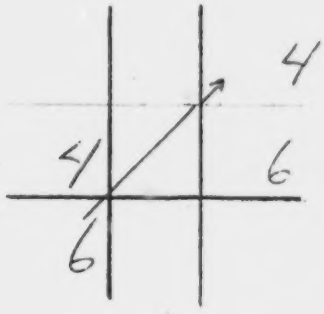


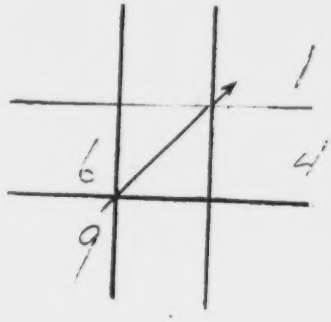
Others need more numbers in place before they can see the patterns. Those students can obtain the number in the third position by adding the numbers in the first two positions.

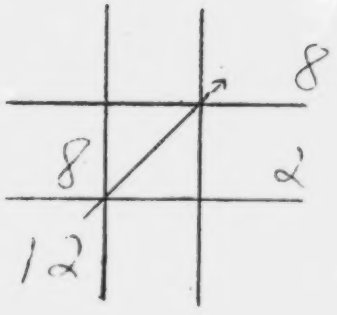


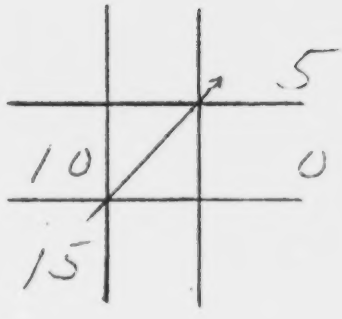
The seventh position now can be calculated by using the pattern of adding to ten. As noted, some students do not need this step because they were able to complete the grid by using the odd number pattern.

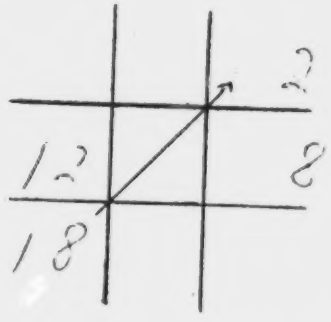


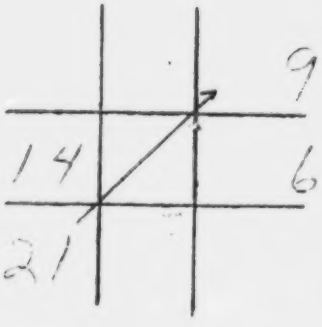


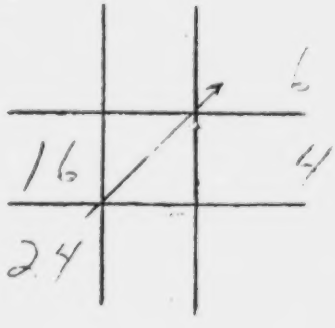


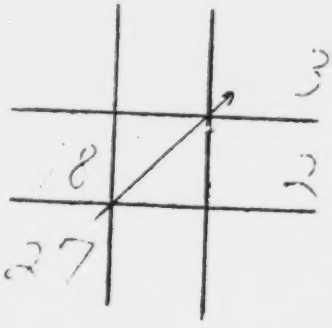












The digits in the one's column for the odd tables now can be finished by counting from one to nine in order.

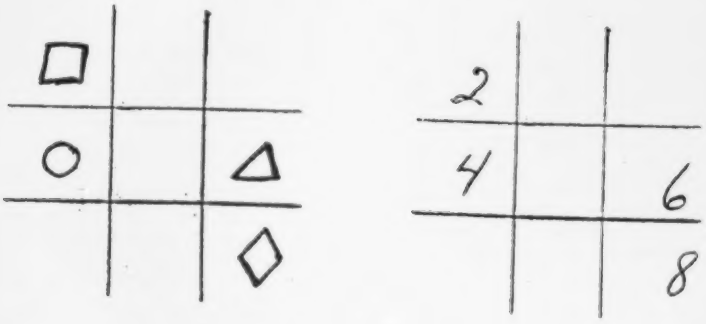
1	4	7
2	5	8
3	6	9

3	2	1
6	5	4
9	8	7

7	8	9
4	5	6
1	2	3

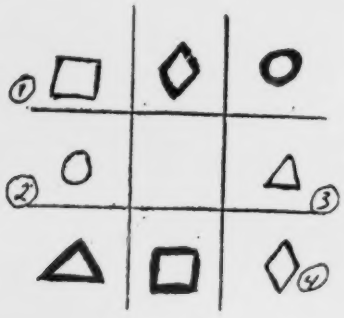
9	6	3
8	5	2
7	4	1

The next step on the even tables is to observe the following pattern.



For some students, it is easier to learn the odd tables first, for others they can learn both the odd and even tables together.

The pattern is completed by using each of the even numbers twice.



2	8	4
4		6
6	2	8

Some students see this quickly, others do not. For those who do not see this pattern easily, try repeating it with the two tables so that they become familiar with it. If this does not work, move to the next optional step.

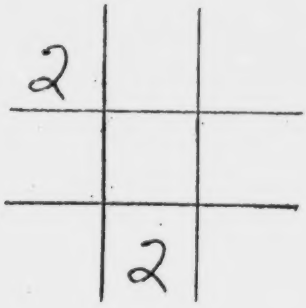
2	8	4
4		
6	2	8

4	6	8
8		2
2	4	6

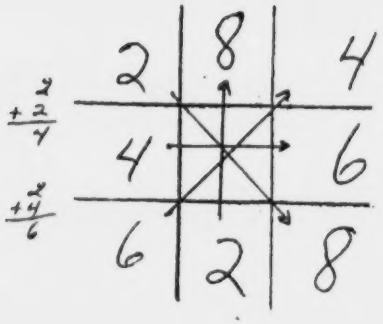
6	4	2
2		8
8	6	4

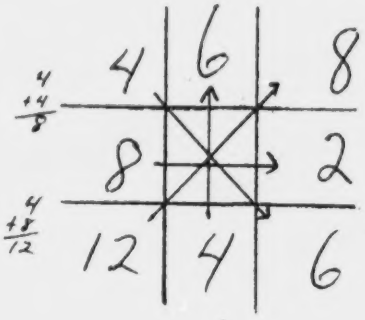
8	2	6
6		4
4	8	2

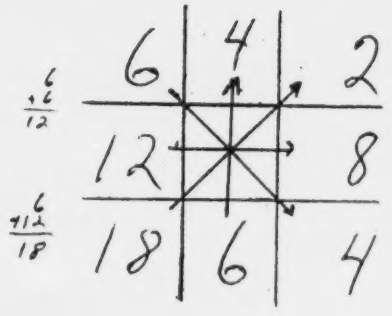
The optional step for the even tables is to begin by placing the number of the table in the first and six positions in the grid before any other numbers.

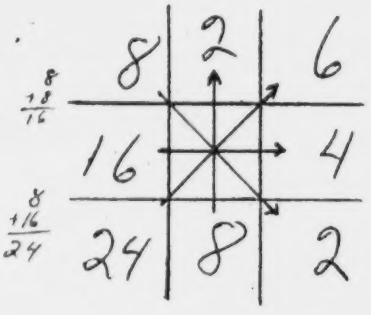


The student then follows the other steps in finding the numbers in the pattern: doubling the first number, adding the first and second numbers, adding the cross and diagonals to ten.









Below are all the tables with the single digits in their appropriate positions.

1	4	7
2	5	8
3	6	9

2	8	4
4	0	6
6	2	8

3	2	1
6	5	4
9	8	7

4	6	8
8	0	2
2	4	6

5	0	5
0	5	0
5	0	5

6	4	2
2	0	8
8	6	4

7	8	9
4	5	6
1	2	3

8	2	6
6	0	4
4	8	2

9	6	3
8	5	2
7	4	1

Once the nine positions have been filled in with the numbers in the ones column (the numbers of the right), the student is ready to complete the tables. Again, some students can learn all of the patterns and tables in one sitting while others will only be able to learn one at a time. Either way it is good to impress upon the student how easily the patterns are used in all the tables.

One method for completing the tables is to have the student increase each number. For example, on the three table the first row lists 3, 6, and 9. The student in asked to think of a number larger than 9 that ends with a 2 and 12 comes to mind, larger than 12 ending in 5 is 15; larger than 15 ending in an 8, 18; larger than 18 ending with 1, 21; larger than 21 ending in a 4, 24; larger than 24 ending in a 7, 27.

3	larger than 9 12	larger than 18 21
6	larger than 12 15	larger than 21 24
9	larger than 15 18	larger than 24 27

This procedure is used on all of the grids, one through nine, resulting in the following completed times tables:

1	4	7
2	5	8
3	6	9

2	8	14
4	10	16
6	12	18

3	12	21
6	15	24
9	18	27

4	16	28
8	20	32
12	24	36

5	20	35
10	25	40
15	30	45

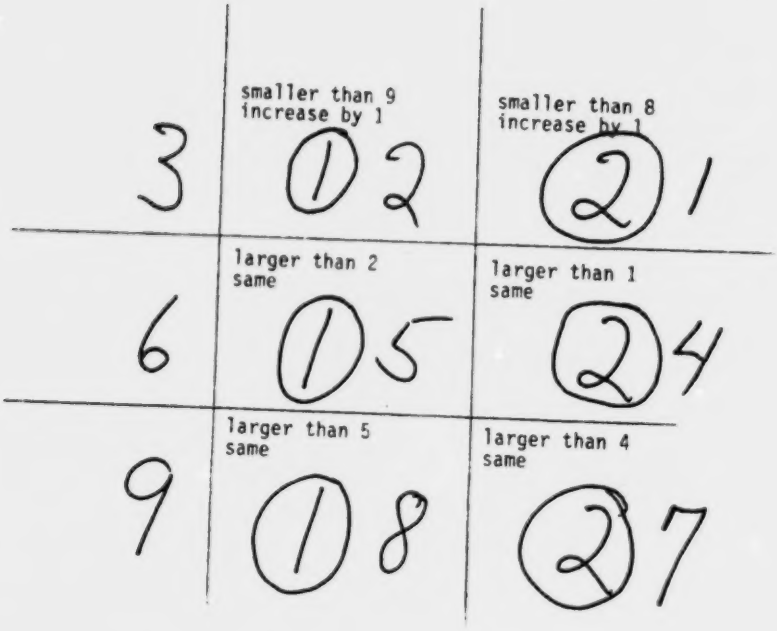
6	24	42
12	30	48
18	36	54

7	28	49
14	35	56
21	42	63

8	32	56
16	40	64
24	48	72

9	36	63
18	45	72
27	54	81

There is an optional method of instructing the completion of the tables. If the number in the one's column in the next position is smaller, then the number in the ten's column is increased by one. If the number in the one's column in the next position is larger, then the number in the ten's column stays the same.



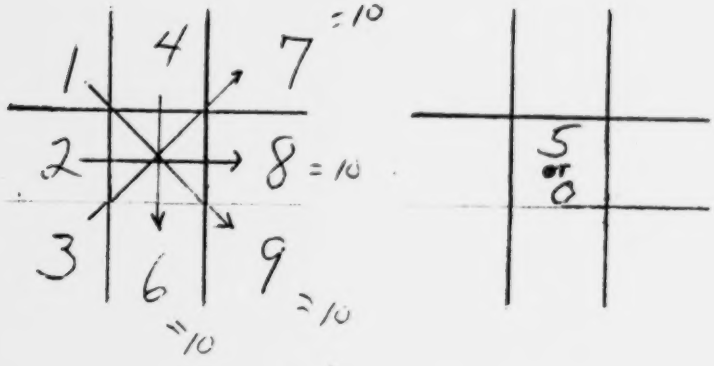
Another method of instructing the completion of the tables is to demonstrate that by adding ten to a number in a position, you can easily calculate the number in the next position because it must be less then the number in the previous position plus 10.

4	add 10 = 22 must be less 16	add 10 = 34 must be less 28
8	add 10 = 26 must be less 20	add 10 = 38 must be less 32
add 10 = 18 must be less 12	add 10 = 26 must be less 24	add 10 = 38 must be less 36

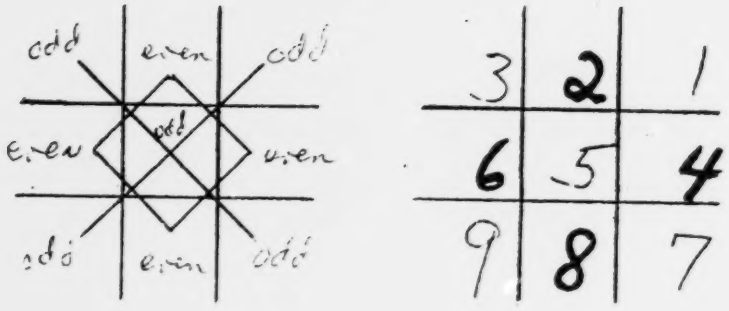
CHECKING THE PATTERNS

The patterns can be checked by making sure that a five or zero is in the center and that the numbers in the ones column in the X and cross patterns add to ten. The odd and even patterns are checked with the X and the diamond. Next, the student can add the full number in each position to check if the numbers add to ten times the first number.

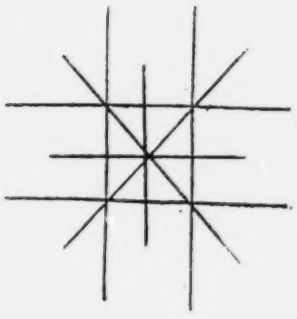
Center is either a five or a zero and numbers in the one's column add to ten:

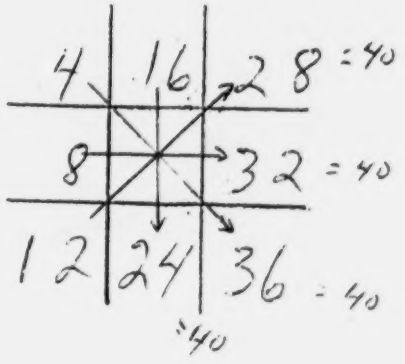


The numbers follow the odd and even patterns on the X and the diamond



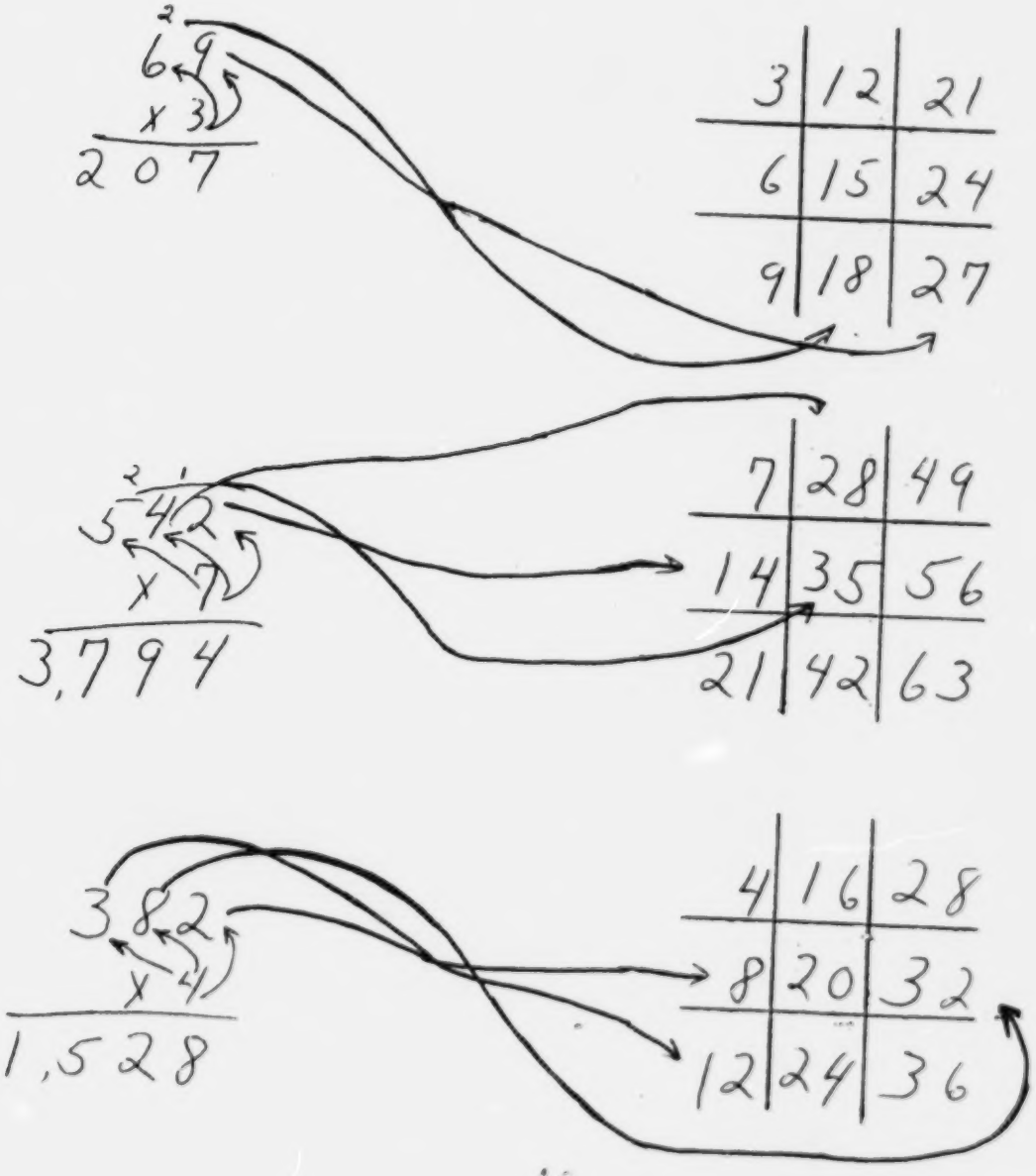
The numbers in the completed grid add to ten times the first number in the table along the X and cross.



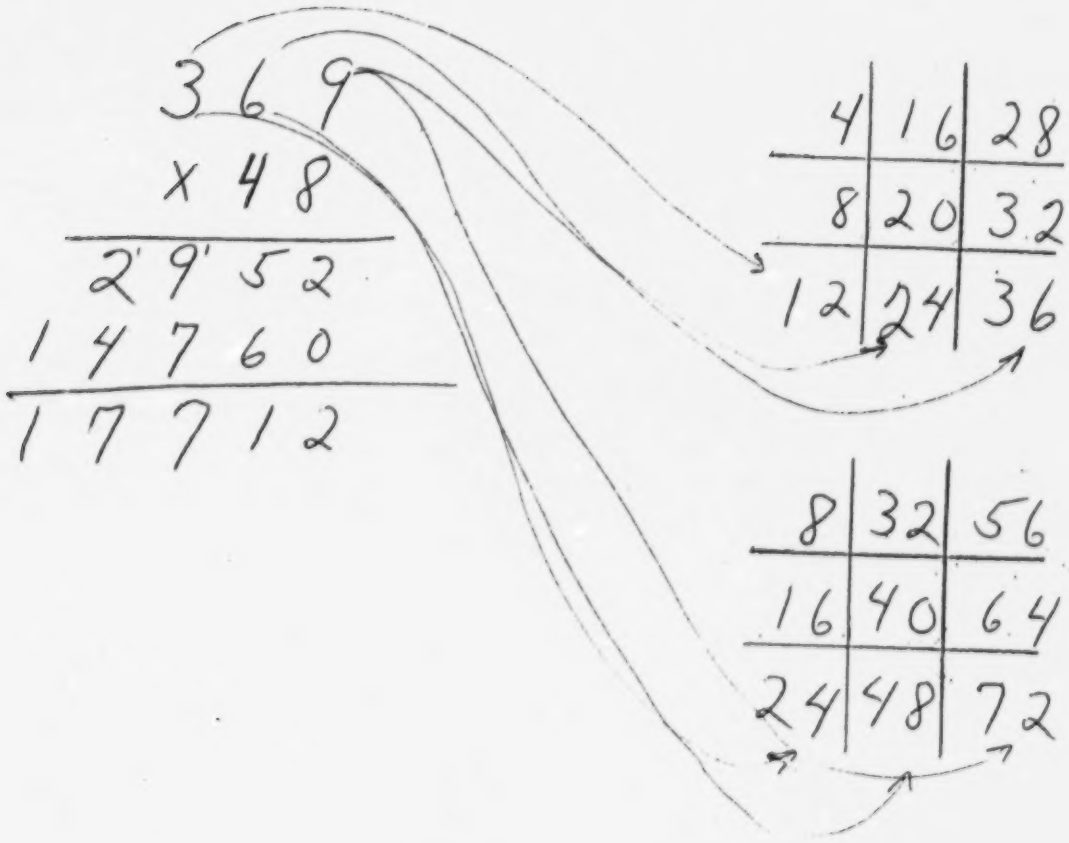


USING THE TIC TAC TOE GRIDS

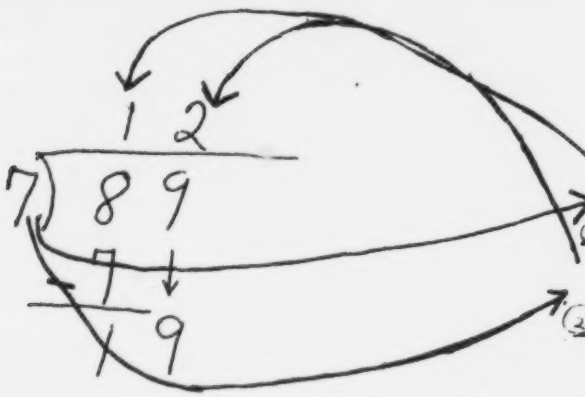
The following is an example of how the grids would be used on a number of simple multiplication problems. Although it appears that there is little difference between having students use the multiplication tables which are printed in sequence and using the Tic Tac Toe grids, there is one large difference. The grids come from the person's memory not from a pre-printed sheet.



The Tic Tac Toe grids provide the student with the ability to complete multiplication problems with a minimum of errors. Problems are easier because the students do not have to be concerned about making a mistake in remembering the tables. Below are some double digit multiplication problems using the grids.



The real beauty of the Tic Tac Toe patterns is in their use for division. The following division problems demonstrate how the patterns remove the step of multiplication from the division process. The concept of "closest without going over" should be taught and reinforced.



7	28	49
14	35	56
21	42	63

```
      5 1 7 1 0 3  R2
   ___________
8 ) 4 1 3 6 8 2 6
  -4 0 ↓
    1 3
   - 8 ↓
    ---
      5 6
    - 5 6 ↓
    -----
        0 8
      - 8 ↓ ↓
      -----
          0 2 6
        - 2 4
        -----
            2
```

8	32	56
16	40	64
24	48	72

Time on task is a basic component of learning. If you practice you improve. The patterns need to be practiced. Each time the student begins working with math, the grids for the nine times tables should be completed. This will reinforce their retention. The optional methods should be practiced also for those students who have memory problems because this provides them with alternative methods for arriving at the answer. If the student refers to previously completed grids, he or she will not obtain sufficient practice and the usefulness of the patterns is reduced. Only through constant use will the student be able to learn the grids adequately for effective use with multiple digit problems.

Another case study about using this technique with an adult with a learning difference (disability) will demonstrated this point. Linda is a 40 year old woman who had never learned her times tables and she had a lot of difficulty with the patterns initially. She could not remember the patterns nor the directions in which the numbers had to be ordered. After six weeks of work (one hour each week), I pointed out to Linda that she had broken a pattern which had prevented her from learning the times tables for 30 years. As she practiced the nine patterns, she froze on one of them. Her old pattern of panic, embarrassment and avoidance overtook her and she said that she could not remember. After a few seconds she look at the pattern again and said "Wait a minute, let me look at this again." Eureka! a pattern broken by the adoption of another. She looked at the patterns and saw that there was more than one way to see them. She completed the table and moved on. It took a number of weeks to overcome her panic, embarrassment and avoidance but overcoming 30 some years of such feelings in six weeks is a good investment of time.

DOUBLE DIGIT GRIDS

The Tic Tac Toe Grids can be used to solve double digit multiplication and division problems. As demonstrated below: the time tables of 10, 20, 30, 40, 50, 60, 70, 80, and 90 are formed by placing a zero behind the completed tables 1 through 9. Note: students may need to be reminded to leave sufficient space in the grids for the extra digit.

10	40	70
20	50	80
30	60	90

20	80	140
40	100	160
60	120	180

30	120	210
60	150	240
90	180	270

40	160	280
80	200	320
120	240	360

50	200	350
100	250	400
150	300	450

60	240	420
120	300	480
180	360	540

70	280	490
140	350	560
210	420	630

80	320	560
160	400	640
240	480	720

90	360	630
180	450	720
270	540	810

The next step in learning the double digits is to add the one tables. The one tables are added behind (or to the right of) the other numbers.

11	44	77
22	55	88
33	66	99

21	84	147
42	105	168
63	126	189

31	124	217
62	155	248
93	186	279

41	164	287
82	205	328
123	246	369

51	204	357
102	255	408
153	306	459

61	244	427
122	305	488
183	366	549

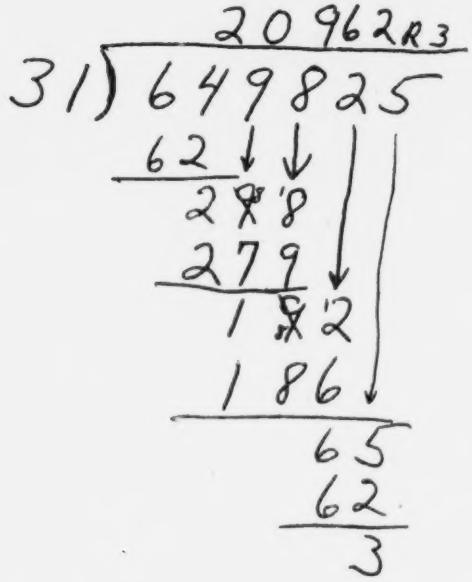
71	284	497
142	355	568
213	426	639

81	324	567
162	405	648
243	486	729

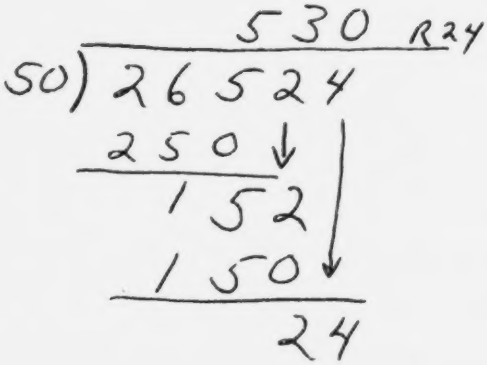
91	364	637
182	455	728
273	546	819

Using Double Digit For Division

Below are examples of double digit division problems



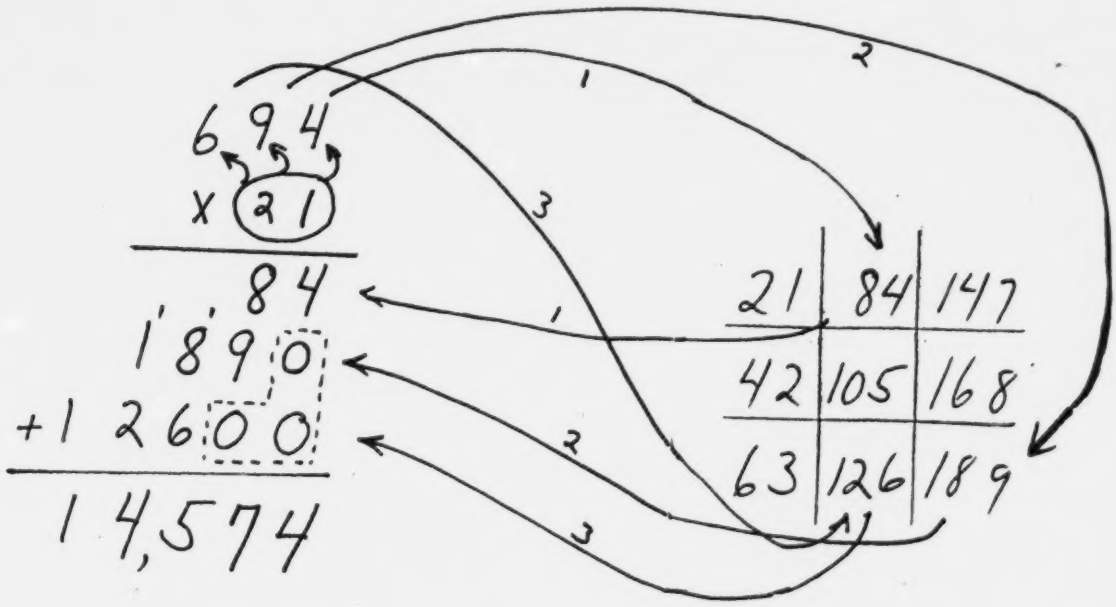
31	124	217
62	155	248
93	186	279



50	200	350
100	250	400
150	300	450

The student is taught to search for the closest number in the grid which does not go over the number in the problem. The number of the position in which this number if found is placed in the answer of the problem and the number in the grid is copied under the last number in the problem.

Below is an alternative method for solving multi-digit multiplication problems. Students who have severe difficulties with carrying numbers are encouraged to use this method.

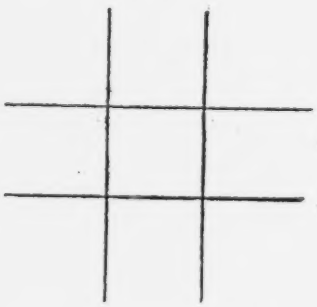


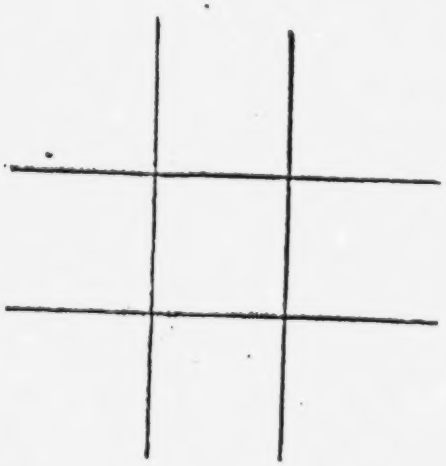
The student needs to be taught to add one zero to the second line before writing numbers and two zeros to the third line. This procedure is continued (adding another zero to the line before writing the number form the grid) for each number being multiplied.

PRODUCING DOUBLE DIGIT MULTIPLICATION TABLES

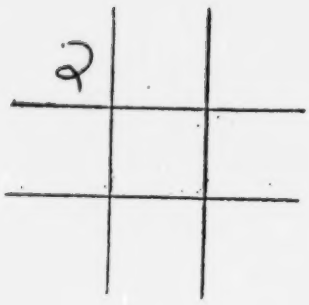
The times tables can be created for every number. Making grids for multi-digit numbers reduces the steps in large multi-digit multiplication and division problems. Since many students with learning differences have problems following multiple directions, this system makes it easier for them. Many students also have problems tracking and placing digits in the appropriate places. As a result, carrying numbers multiple times increases errors. Carrying a number of times to the same number is eliminated in this system and the steps for doing division are greatly reduced.

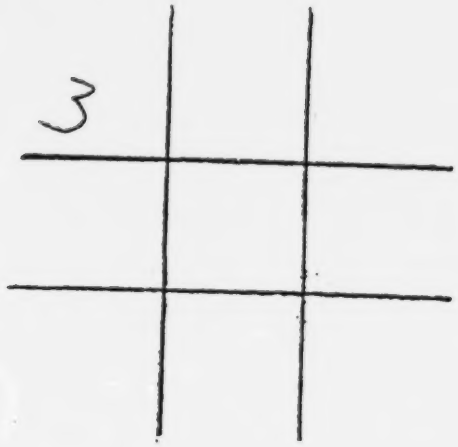
Once the Tic Tac Toe grids are learned, creating grids for any number is a simple procedure. It requires only basic addition. Two digit grids ending in zero or one have already been discussed. Students should be encouraged to form their numbers neatly so they can read the grids. When creating two digit grids, the second one should be a little larger or the numbers should be smaller.





Let us start with the number 23. Create two grids:





Complete the basic grids for both with the number in the one's column and the full grid for the grid on the left. The grid on the right should be completed as follows with the numbers in the ten's column written small.

2	8	14
4	10	16
6	12	18

3	,2	₂1
6	,5	₂4
9	,8	₂7

These numbers are written small because they are eliminated in the grid when the two are combined.

Next the grids are combined by moving the number in the left grid to the corresponding position in the right.

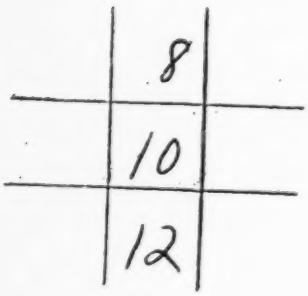
2	8	14
4	10	16
6	12	18

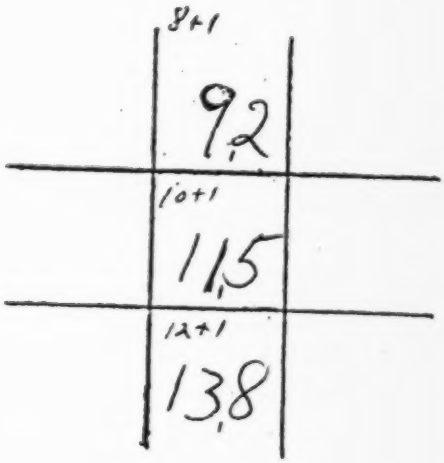
23	,2	₂1
46	,5	₂4
69	,8	₂7

This results in 1 x 23 = 23
2 x 23 = 46
3 x 23 = 69

This was a simple step which is easily learned. The next column in the right grid has small numbers in front of the large numbers. The number being brought over is added to the small number. In the example below the number eight is being moved over to the fourth position and it is added to the small one, resulting in 9. 92 is now in the fourth place because 4 x 23 = 92. The procedure is repeated for the fifth and sixth positions.

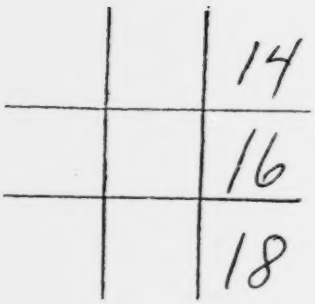
so 5 x 23 = 115
and 6 x 23 = 138

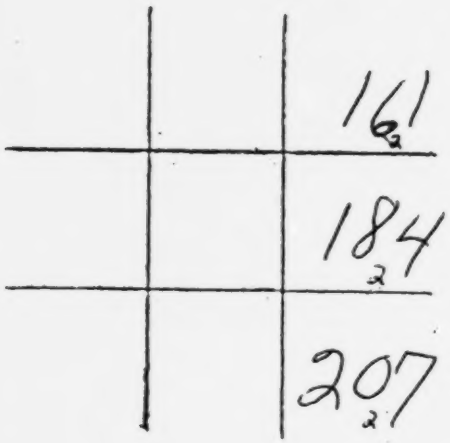




The numbers in the last three positions are brought over in a similar manner. This time there is a small two in each position so two is added to each number. Therefore 14 becomes 16, 16 is moved over and becomes 18 and 18 is moved over and becomes 20.

so
7 x 23 = 161
8 x 23 = 184
9 x 23 = 207



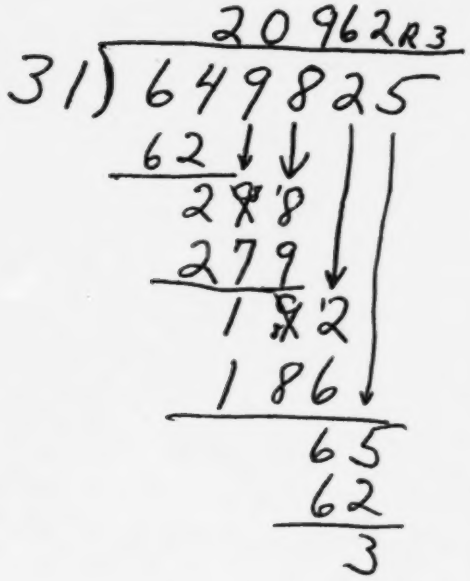


The completed grids look like those shown below. The small numbers are either erased or written over.

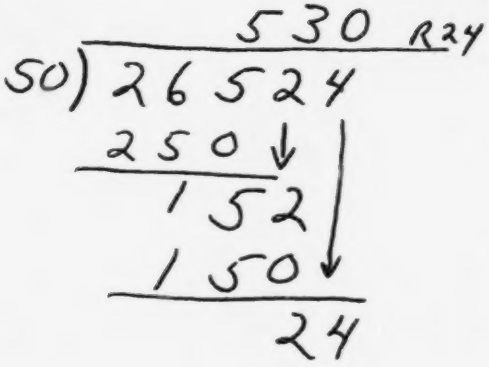
23	92	161
46	115	184
69	138	207

Using Double Digit For Division

Below are examples of double digit division problems



31	124	217
62	155	248
93	186	279



50	200	350
100	250	400
150	300	450

The student is taught to search for the closest number in the grid which does not go over the number in the problem. The number of the position in which this number if found is placed in the answer of the problem and the number in the grid is copied under the last number in the problem.

Below is another example of a double digit problem:

9	36	63
18	45	72
27	54	81

98	39,2	68,6
19,6	49,0	78,4
29,4	58,8	88,2

$$\begin{array}{r} 942 \\ \times\ 98 \\ \hline 1'96 \\ 3'920 \\ +\ 8'82\,00 \\ \hline 92,316 \end{array}$$

$$\begin{array}{r} 475\ R\ 48 \\ 98\overline{)37'65'98} \\ -392\downarrow\ \ \ \ \\ \hline 1'73\ 9\ \ \\ -686\ \ \ \\ \hline 1'05\ 38 \\ 490\ \ \\ \hline 48 \end{array}$$

PRODUCING TRIPLE DIGIT MULTIPLICATION TABLES

Three digit numbers can be formed in a similar way. If the number is 230 or 231, simply add zeros or the one times tables to the numbers in same grid if it is big enough or create a third grid.

230	920	1610
460	1150	1840
690	1380	2070

231	924	1617
462	1155	1848
693	1386	2079

If the number does not end in a zero or one, then a third grid is required. The numbers in the second grid are combined with the numbers in the third grid. No number larger than 8 ever appears as a small number. Let us create a grid for 259.

2	8	14
4	10	16
6	12	18

25	10$_2$0	17$_3$5
5,0	12$_2$5	20,0
7,5	15$_3$0	22$_4$5

259	103$_3$6	181,3
51,8	129$_4$5	207,2
77,7	155$_5$4	233$_8$1

Below is an example of solving triple digit multiplication and division problems with the triple digit grids.

$$
\begin{array}{r}
6482 \\
\times\ 259 \\
\hline
5\,18 \\
20'720 \\
1036\,00 \\
1554\,000 \\
\hline
1,678,838
\end{array}
$$

$$
\begin{array}{r}
365\ R3 \\
259\overline{)\ 9538} \\
-777\downarrow \\
\hline
1613 \\
-1554\downarrow \\
\hline
1298 \\
-1295 \\
\hline
3
\end{array}
$$

259	103,6	181.3
51.8	129.5	207.2
77.7	155.4	233.1

PRODUCING MULTI-DIGIT TABLES

It should now be apparent that a grid can be created for any number using the same procedure. This makes multiple digit multiplication and division simple as is observed in the following example of 54,038 x 5,493,873 or 987,965,733 divided by 54,038

5	20	35
10	25	40
15	30	45

54	216	378
108	270	432
162	324	486

540	2160	3780
1080	2700	4320
1620	3240	4860

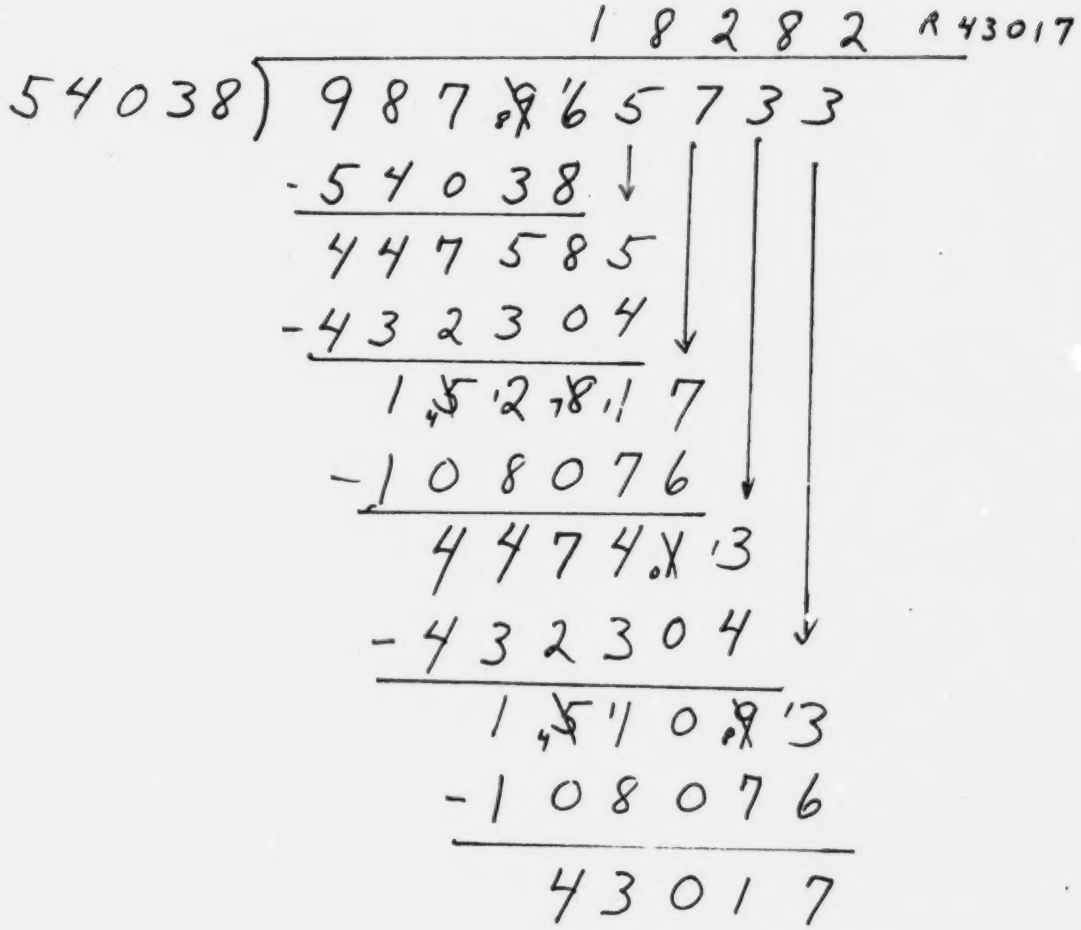
5403	21612	37821
10806	27015	43224
16209	32418	48627

54038	216152	378266
108076	270190	432304
162114	324228	486342

58

$$
\begin{array}{r}
5,493,873 \\
\times\ 54038 \\
\hline
1^{2}62'114 \\
'3782660 \\
\ '432304\overset{1}{0}0 \\
\ ^{2}162114000 \\
'486342000\overset{1}{0} \\
21615200000 \\
270190000000 \\
\hline
296,877,909,174
\end{array}
$$

54038	2161.52	37826.6
108076	270190	43230.4
162114	324228	48634.2



54038	2161.52	378266
108076	270190	432304
162114	324228	486342

The more a student uses the grids the faster he or she becomes. Large multiplication and long division problems become simple tasks and the student who previously avoided math can develop a new sense of self-confidence. This new self-confidence can begin a new pattern of success and achievement. Good luck and remember "I MIGHT LEARN DIFFERENTLY BUT I LEARN WELL."

Tic Tac Toe Math

Workbook I

By

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ODD EVEN EXERCISE

1. Place an E after the even numbers and an O after the odd numbers.

1	_____	2	_____	3	_____	4	_____	5	_____
6	_____	7	_____	8	_____	9	_____	10	_____
11	_____	12	_____	13	_____	14	_____	15	_____
16	_____	17	_____	18	_____	19	_____	20	_____
21	_____	22	_____	23	_____	24	_____	25	_____
26	_____	27	_____	28	_____	29	_____	30	_____
31	_____	32	_____	33	_____	34	_____	35	_____
36	_____	37	_____	38	_____	39	_____	40	_____
41	_____	42	_____	43	_____	44	_____	45	_____
46	_____	47	_____	48	_____	49	_____	50	_____

ODD EVEN EXERCISE

1. Place an E after the even numbers and an O after the odd numbers.

3 _____	9 _____	33 _____	18 _____	5 _____
26 _____	48 _____	25 _____	19 _____	11 _____
99 _____	56 _____	73 _____	71 _____	80 _____
44 _____	37 _____	97 _____	89 _____	50 _____
73 _____	67 _____	22 _____	77 _____	52 _____
162 _____	278 _____	475 _____	299 _____	501 _____
105 _____	321 _____	432 _____	989 _____	352 _____
760 _____	872 _____	583 _____	799 _____	790 _____
547 _____	428 _____	731 _____	444 _____	910 _____
363 _____	705 _____	282 _____	397 _____	501 _____

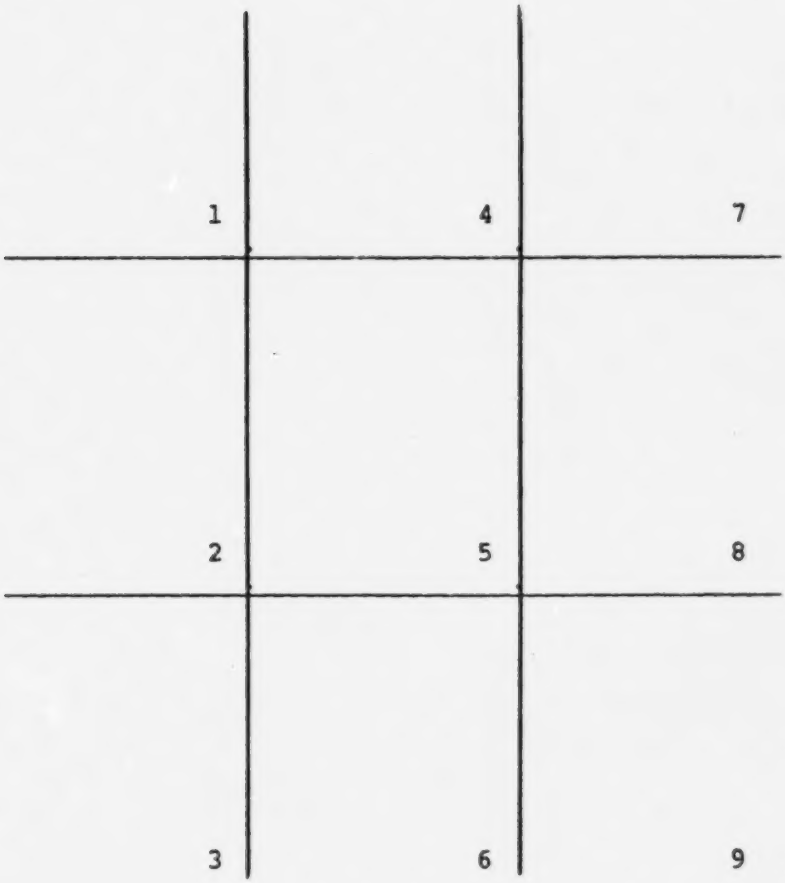
ADDITION EXERCISE

1. Complete the problems.

5 +5	6 +4	7 +3	8 +2	9 +1
8 +8	6 +6	7 +7	5 +5	9 +9
5 +5	1 +1	3 +3	4 +4	2 +2
4 +6	2 +8	5 +5	1 +9	3 +7
1 +1	6 +6	3 +3	8 +8	9 +1
5 +5	1 +1	3 +3	4 +4	2 +2
4 +6	2 +8	5 +5	1 +9	3 +7
6 +6	7 +7	5 +5	9 +9	8 +8

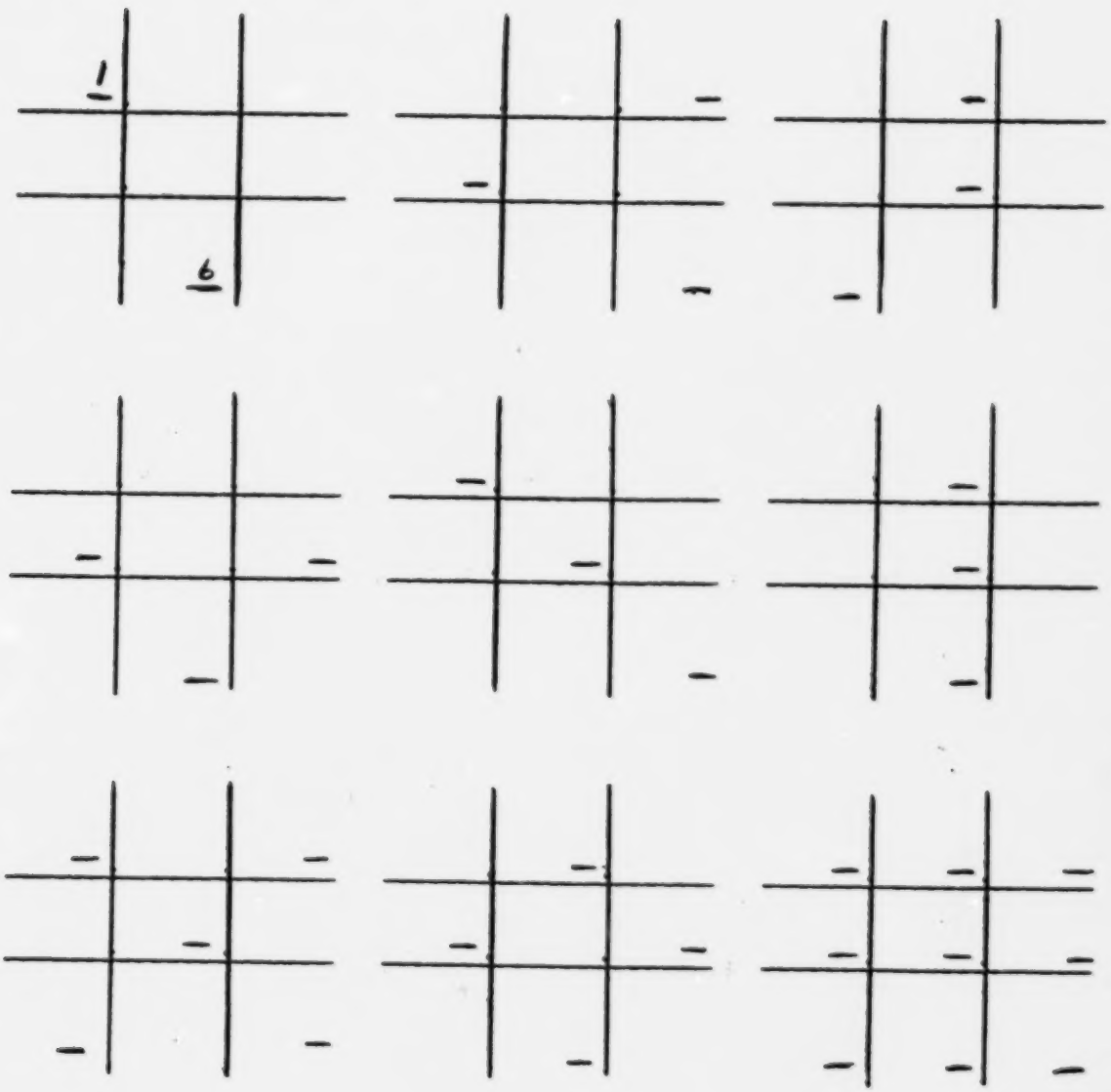
THE POSITIONS

The positions or boxes of the Tic Tac Toe grids are named by counting down. Notice the first grid. The names of the positions are written in the corners. (You will notice that the names are the same as the one times tables.)



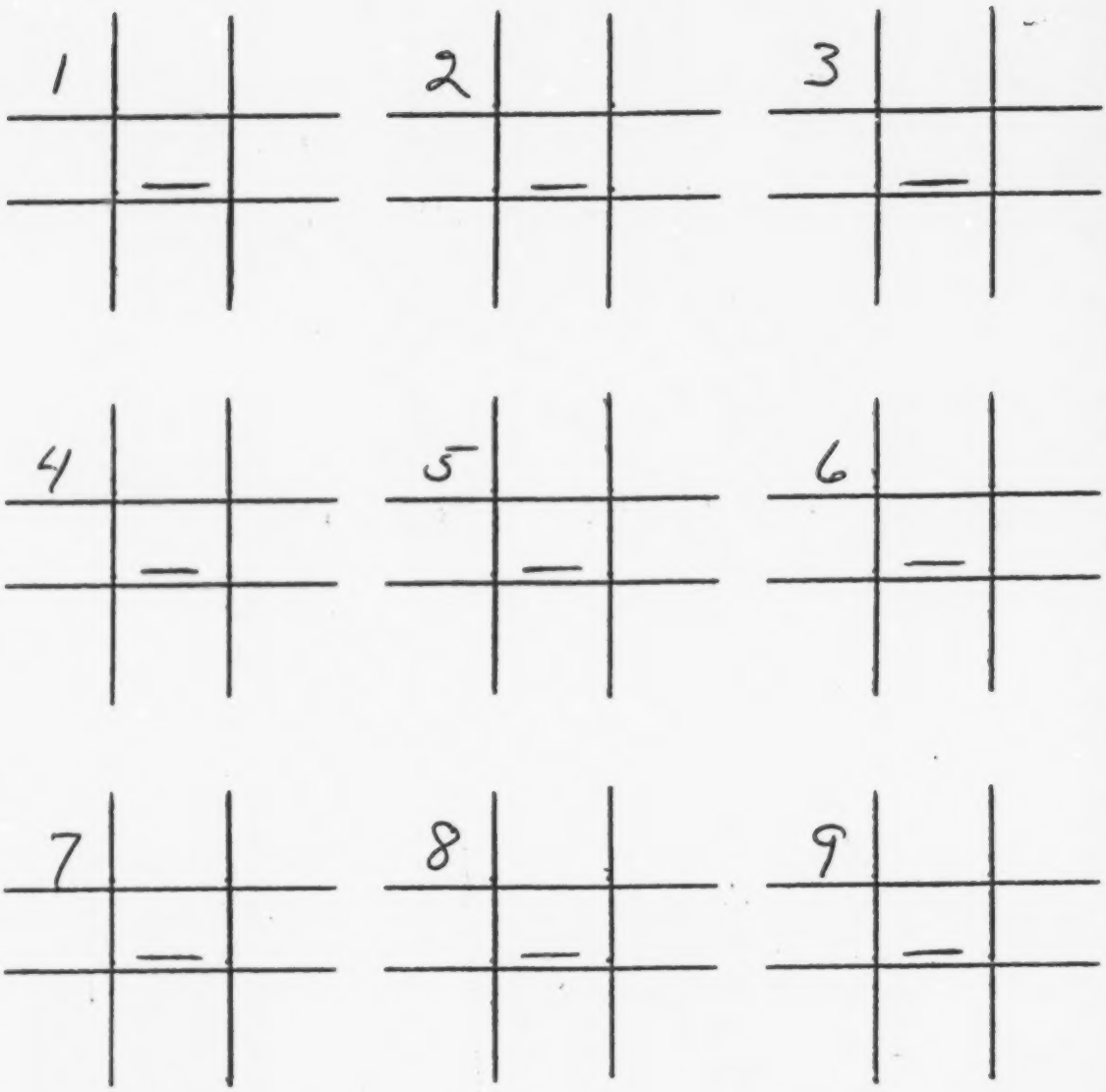
NAME THE POSITIONS

Place the number which names the positions on the lines in each of the grids.



RULE 1

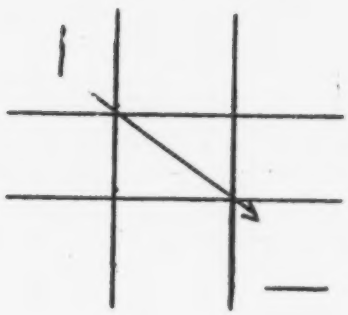
If the table is odd there is a 5 in the center position or if the table is even there is a 0 in the center position.



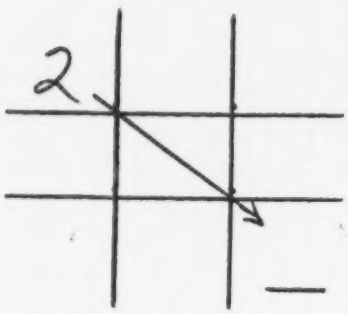
RULE 2

All opposite positions add to 10 (diagonally and across). The five in the center position on the odd table is not counted.

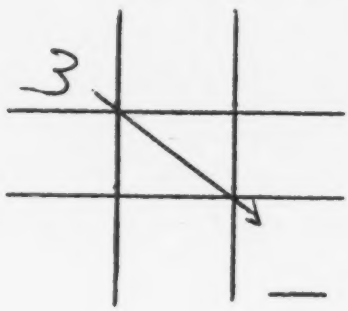
1 + ? = 10



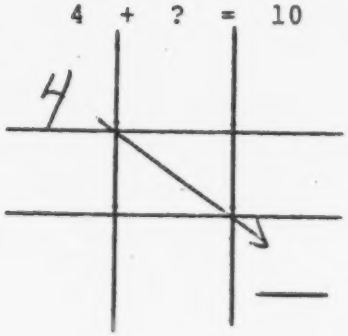
2 + ? = 10



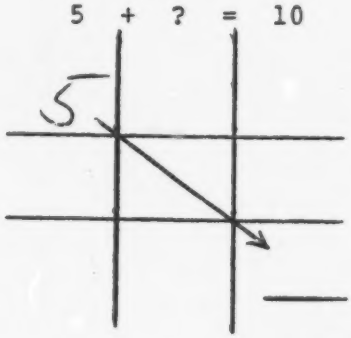
3 + ? = 10



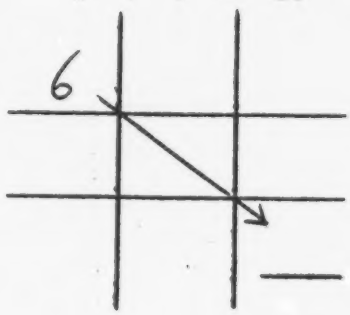
4 + ? = 10



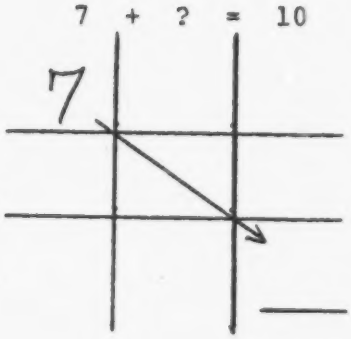
5 + ? = 10



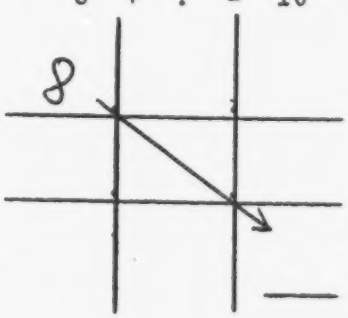
6 + ? = 10



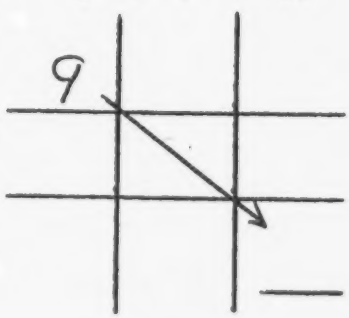
7 + ? = 10



8 + ? = 10



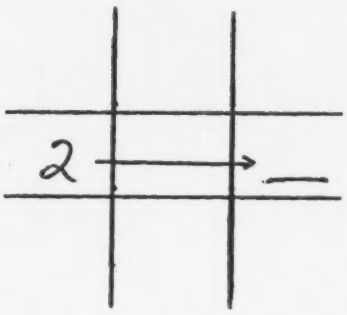
9 + ? = 10



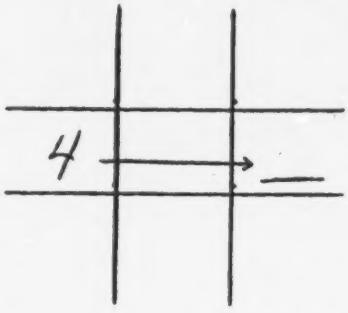
RULE 2

Try it again, this time from the second position. The eighth position will add to 10 with the second position.

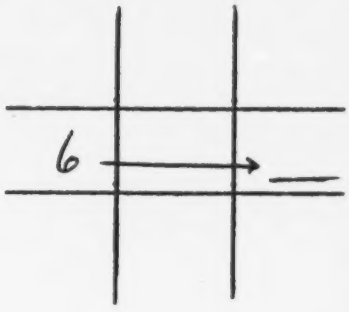
2 + ? = 10



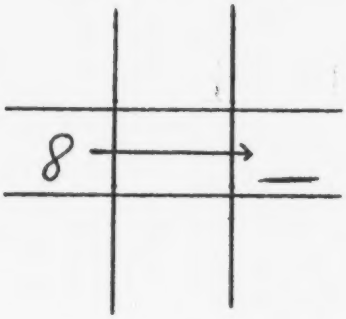
4 + ? = 10



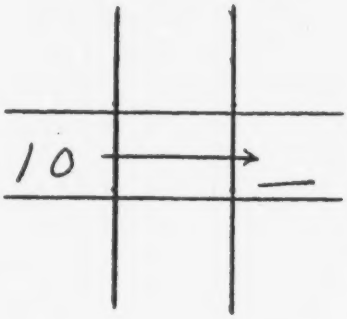
6 + ? = 10



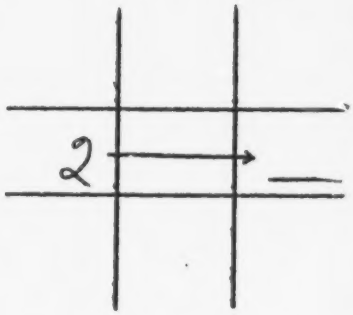
8 + ? = 10



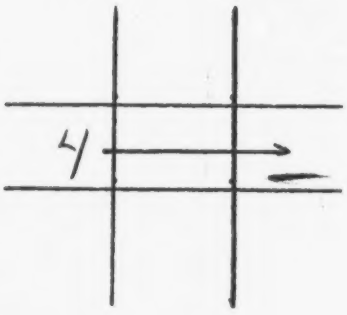
10 + ? = 10



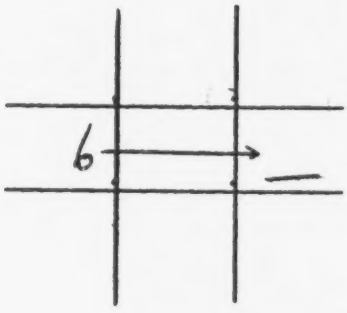
2 + ? = 10



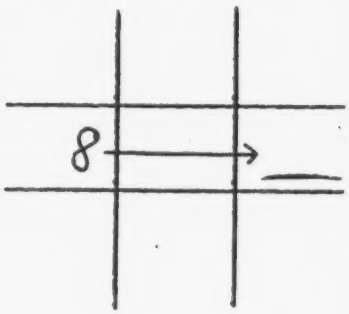
4 + ? = 10



6 + ? = 10



8 + ? = 10

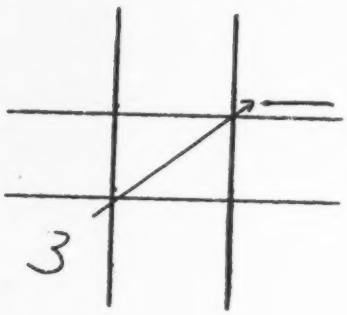


8

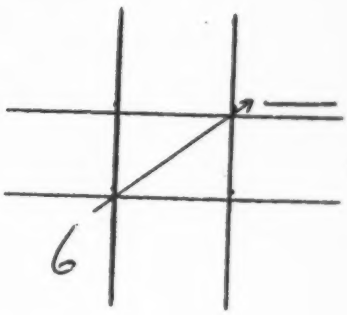
RULE 2

Try it again, this time from the third position. The seventh position will add to 10 with the third position.

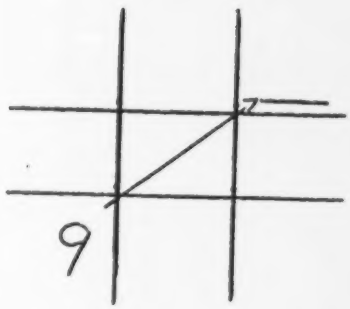
3 + ? = 10



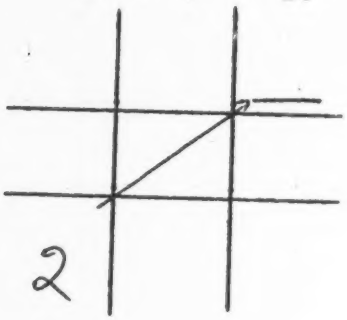
6 + ? = 10



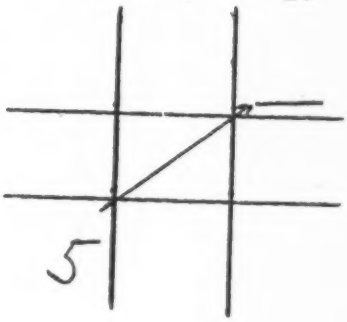
9 + ? = 10



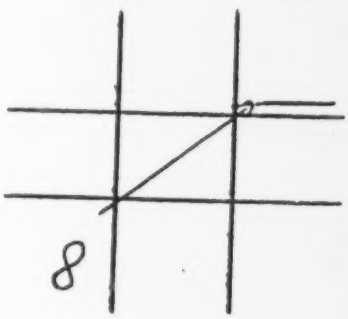
2 + ? = 10



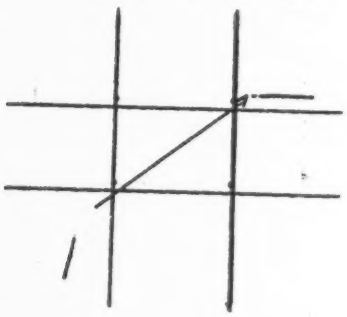
5 + ? = 10



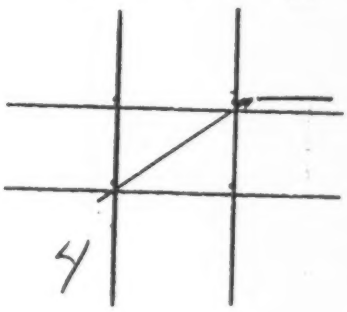
8 + ? = 10



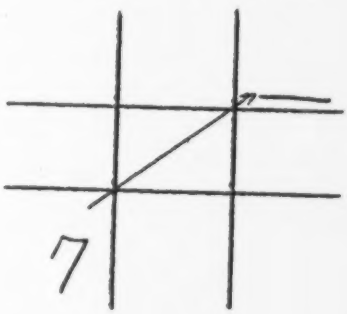
1 + ? = 10



4 + ? = 10



7 + ? = 10

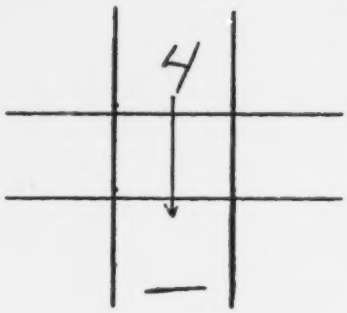


9

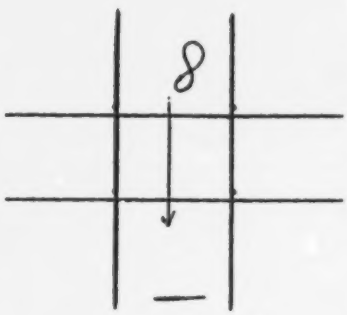
RULE 2

Let's do it one more time. The fourth position will add to 10 with the sixth position.

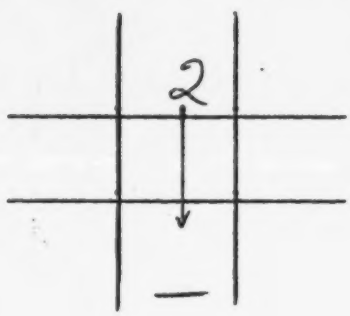
4 + ? = 10



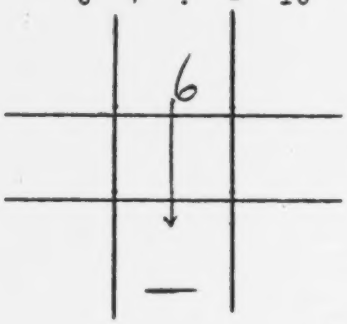
8 + ? = 10



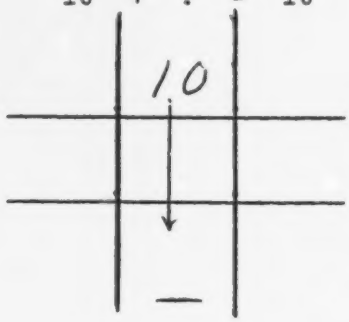
2 + ? = 10



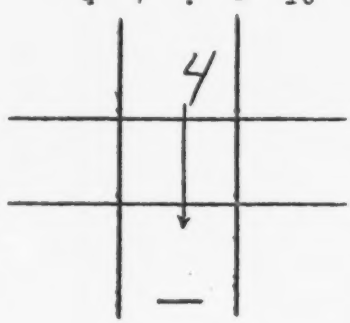
6 + ? = 10



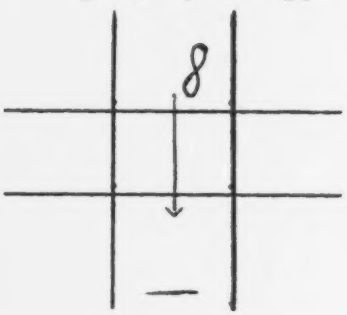
10 + ? = 10



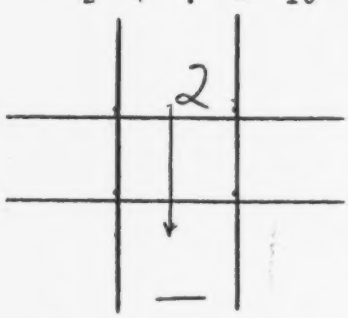
4 + ? = 10



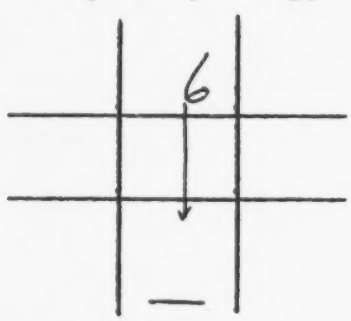
8 + ? = 10



2 + ? = 10



6 + ? = 10

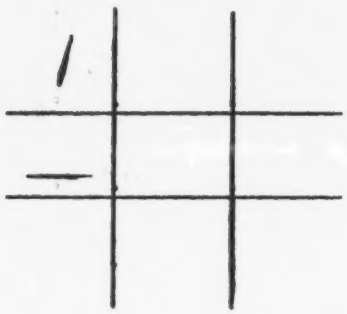


10

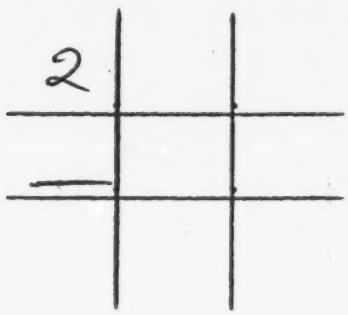
THE NUMBER IN THE SECOND POSITION

The number in the second position is obtained by doubling the number in the first position. (For example: 1 + 1 or 8 + 8.)

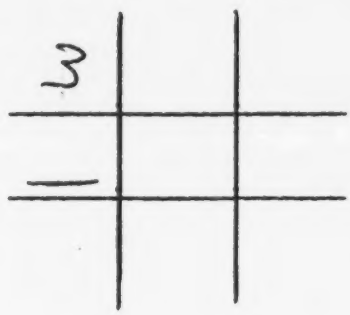
1 + 1 = ?



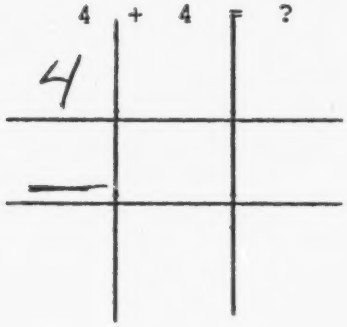
2 + 2 = ?



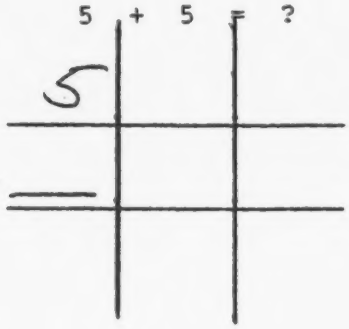
3 + 3 = ?



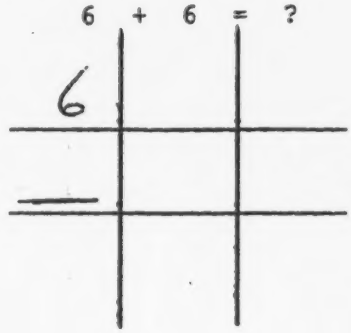
4 + 4 = ?



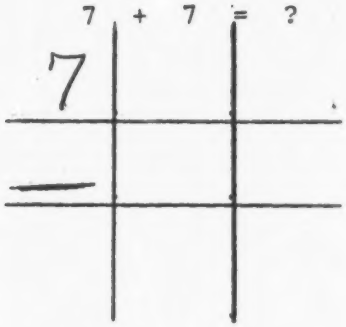
5 + 5 = ?



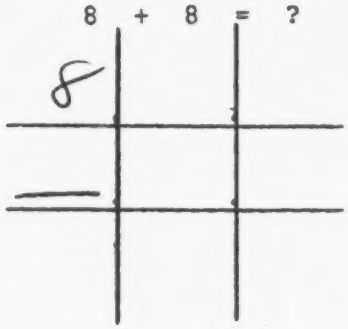
6 + 6 = ?



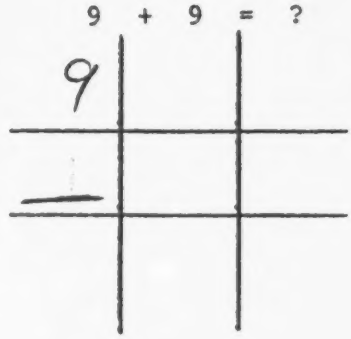
7 + 7 = ?



8 + 8 = ?

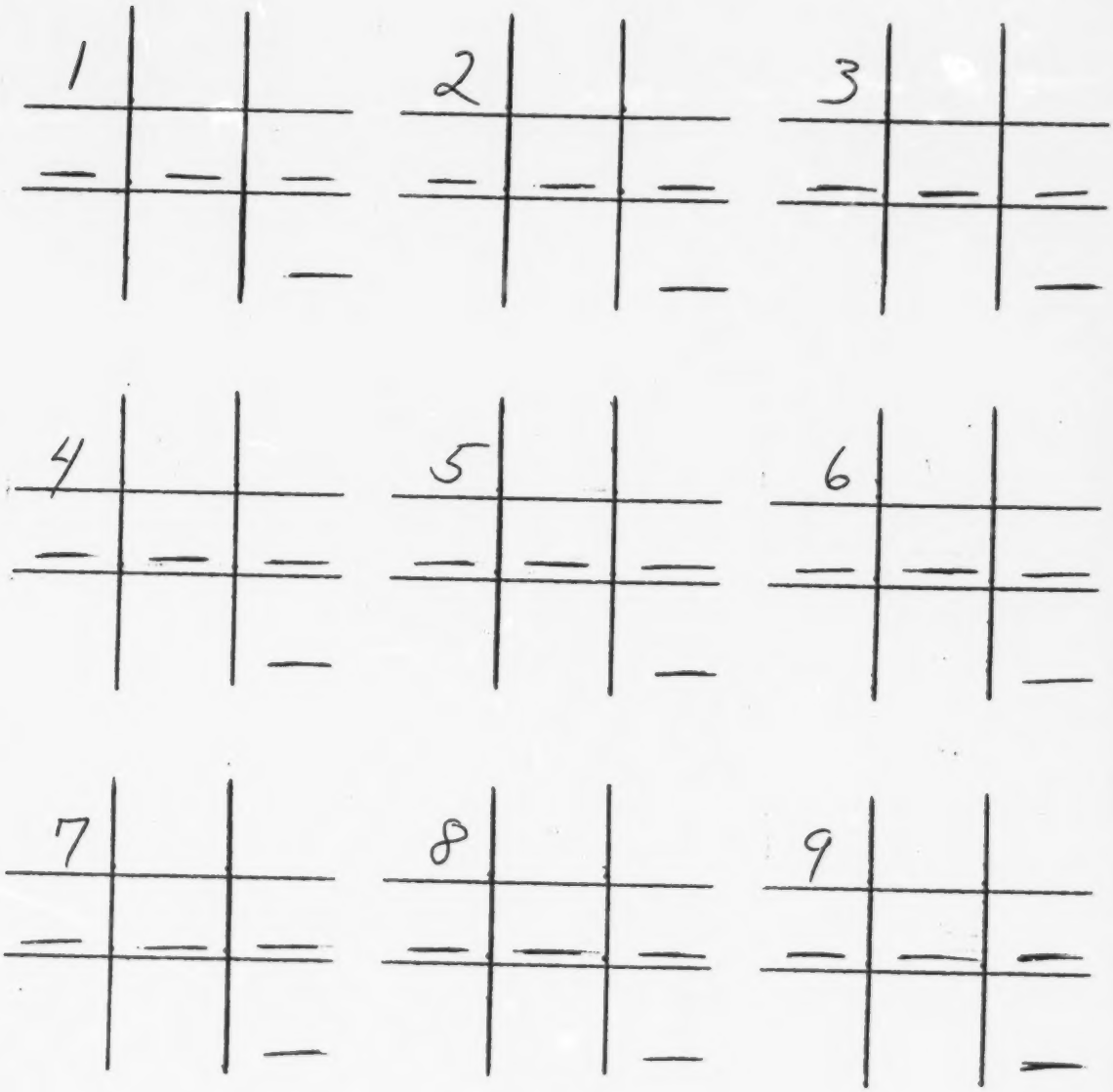


9 + 9 = ?



STARTING THE GRIDS

Now it is time to fill in the positions using rules one and two. Start by placing the correct number in the center position. Next fill in the second, eighth and ninth positions.



THE ODD PATTERN

The times tables of the odd numbers 1, 3, 7 and 9 have a very noticeable pattern -- each number from 1 to 9 is in order in these grids. You will see that the direction is different for each grid. To fill in the missing numbers of the grid you need only count. Look at the 1 time tables. You begin counting 1, 2 and fill in 3 and 4 in the third and fourth positions. The 5 is already there. You place a 6 and 7 in the sixth and seventh positons. The 8 and 9 are already there.

Now try the 3 times tables. Begin counting with the 4. You are counting from right to left 4, 5, 6, and 7. Place an 8 and 9 in the sixth and third position. In the seventh position, diagonally from the third, you place a 1 (Rule 2) and a 2 in the fourth position.

Look at the 7 times tables. You see that there is a 14 in the second position. Ignore the 1 and only consider the 4. Some students cover the one with their finger so they do not get confused. Begin counting left to right, 4, 5, 6 and 7. Place an 8 and 9 in the fourth and seventh positions. Again use Rule 2 to place a 1 in the third position (opposite positions add to 10) and place a 2 in the fourth position.

The 9 times tables can be either counted backwards 9, 8, placing a 7 in the third position, etc. or by counting 1, 2, placing a 3 in the seventh position.

1	—	—
2	5	8
—	—	9

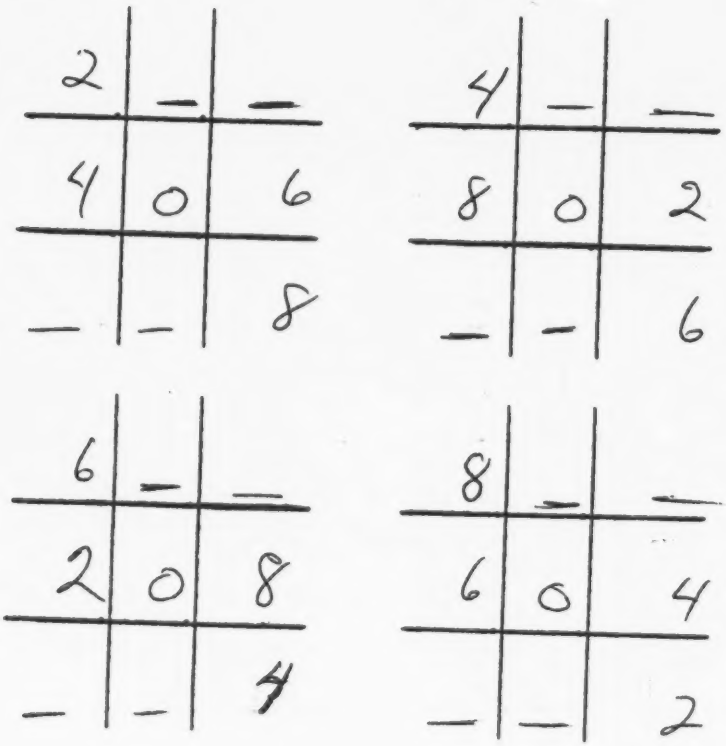
3	—	—
6	5	4
—	—	7

7	—	—
14	5	6
—	—	3

9	—	—
18	5	2
—	—	1

THE EVEN PATTERN

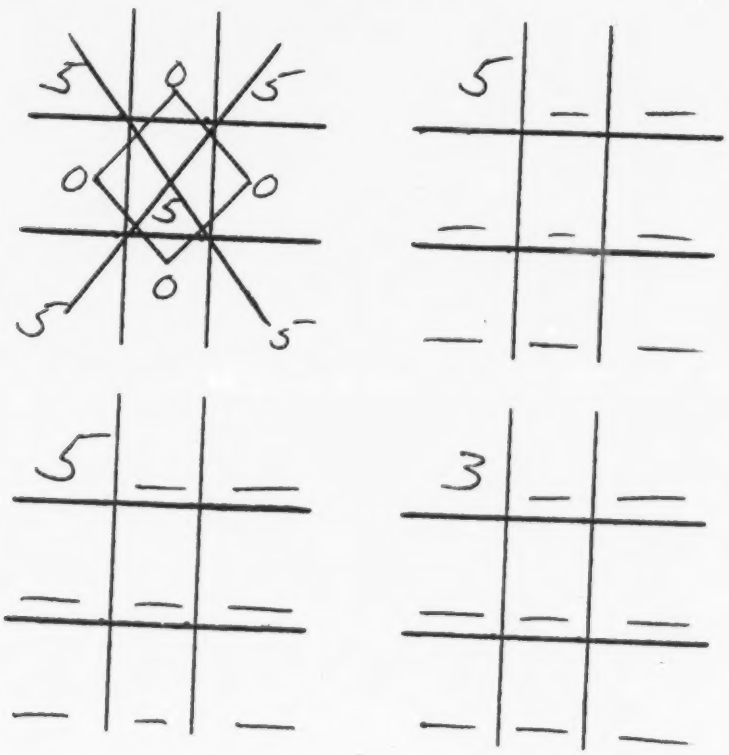
The even pattern is also very noticeable. There are no odd numbers in the even tables. Each even number 2, 4, 6 and 8 are in the grid twice. To complete the even grid repeat the numbers that are already in the grid. For example 2, 4, 6, 8. Now repeat this pattern twice filling in the positions as required. Another way to complete the even grids is to begin by placing the number of the times table in both the first and sixth position. Then use Rule 2 (all opposite positions add to 10) to complete the grid. If you have difficulty with the even pattern, ask your instructor for further explanation.



THE X AND DIAMOND PATTERN

The X and diamond pattern is used to create 5 times table and to check the odd times tables. In the 5 times tables, there is a 5 in each position along the X and a 0 in the positions touched by the diamond.

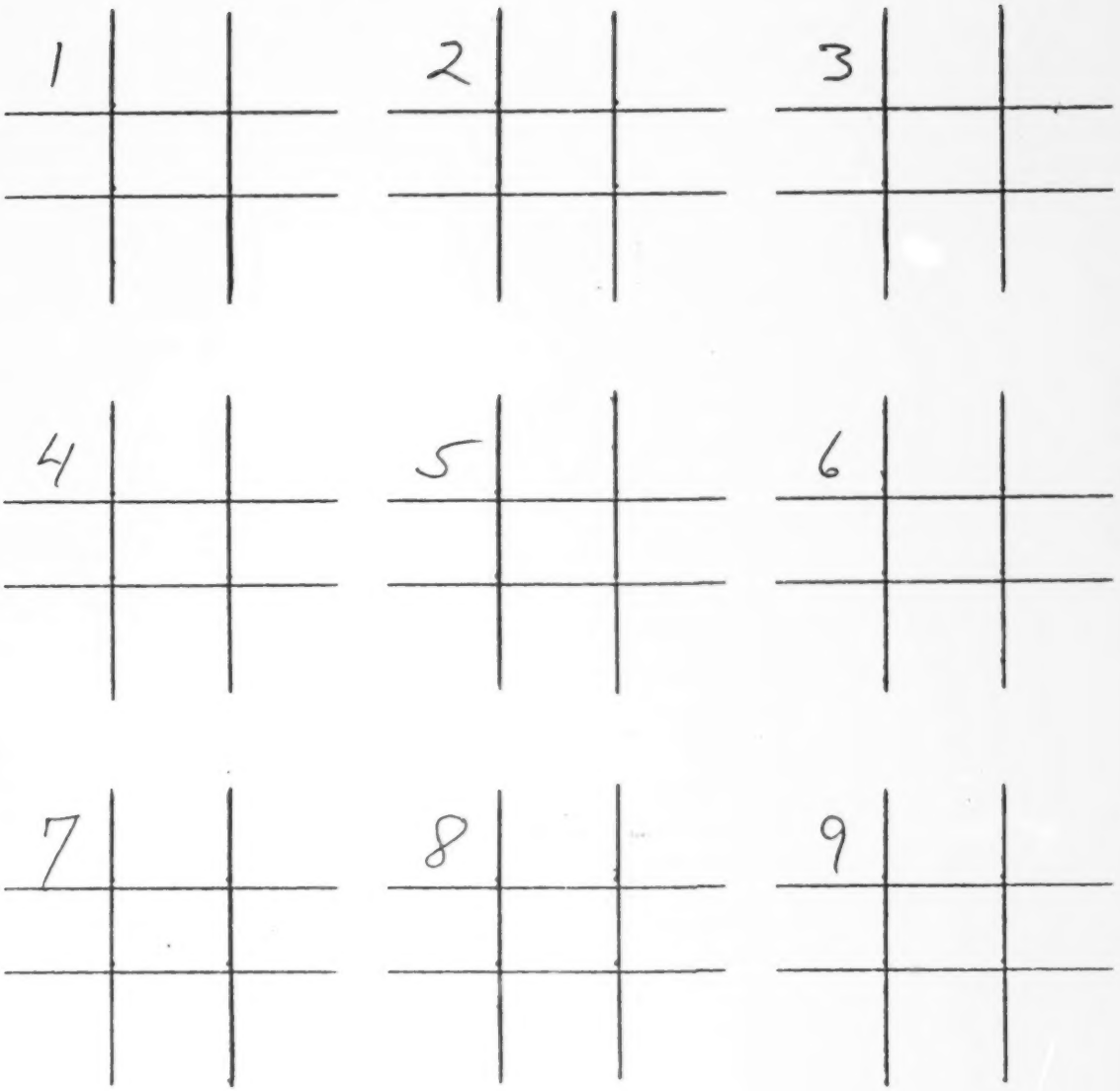
You can check the odd time table grids the same way. There are odd numbers in each positions along the X and even numbers in the positions touched by the diamond.



Now draw an X and a diamond over your grids to check to see if there are odd numbers on the X and even numbers on the diamond.

COMPLETING THE TABLES

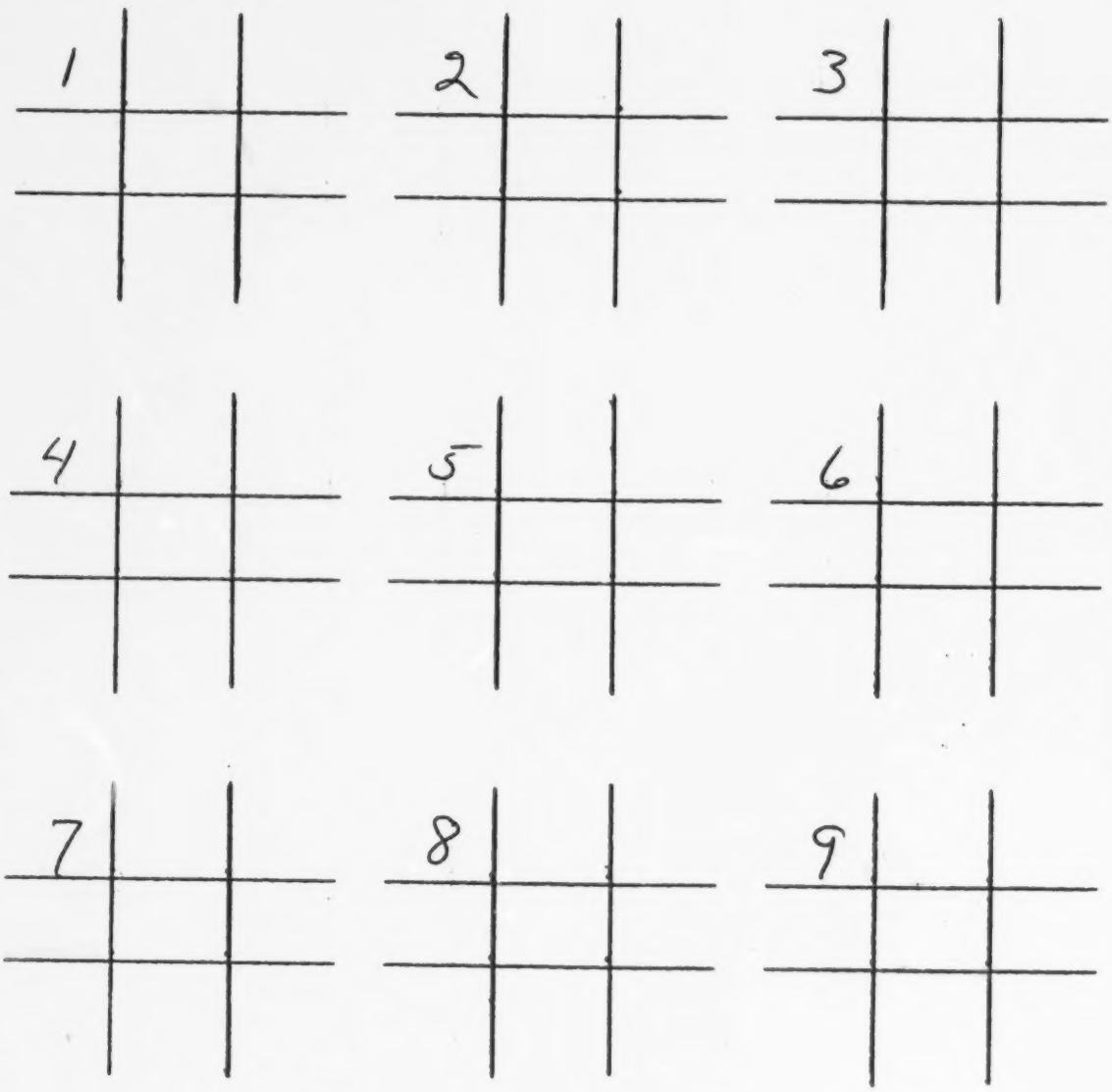
Before we move on, let us practice completing the grids.



After you finish filling in the positions in all of the grids, you can complete the times table by placing a number in the ten column (in front of the number in the grid). The 1 time table is complete. The 2 times table is completed by placing a 1 in front of the 0 in the fifth position, a 1 in front of the 2 in the sixth position etc. Try it; if you have difficulty, consult with your instructor.

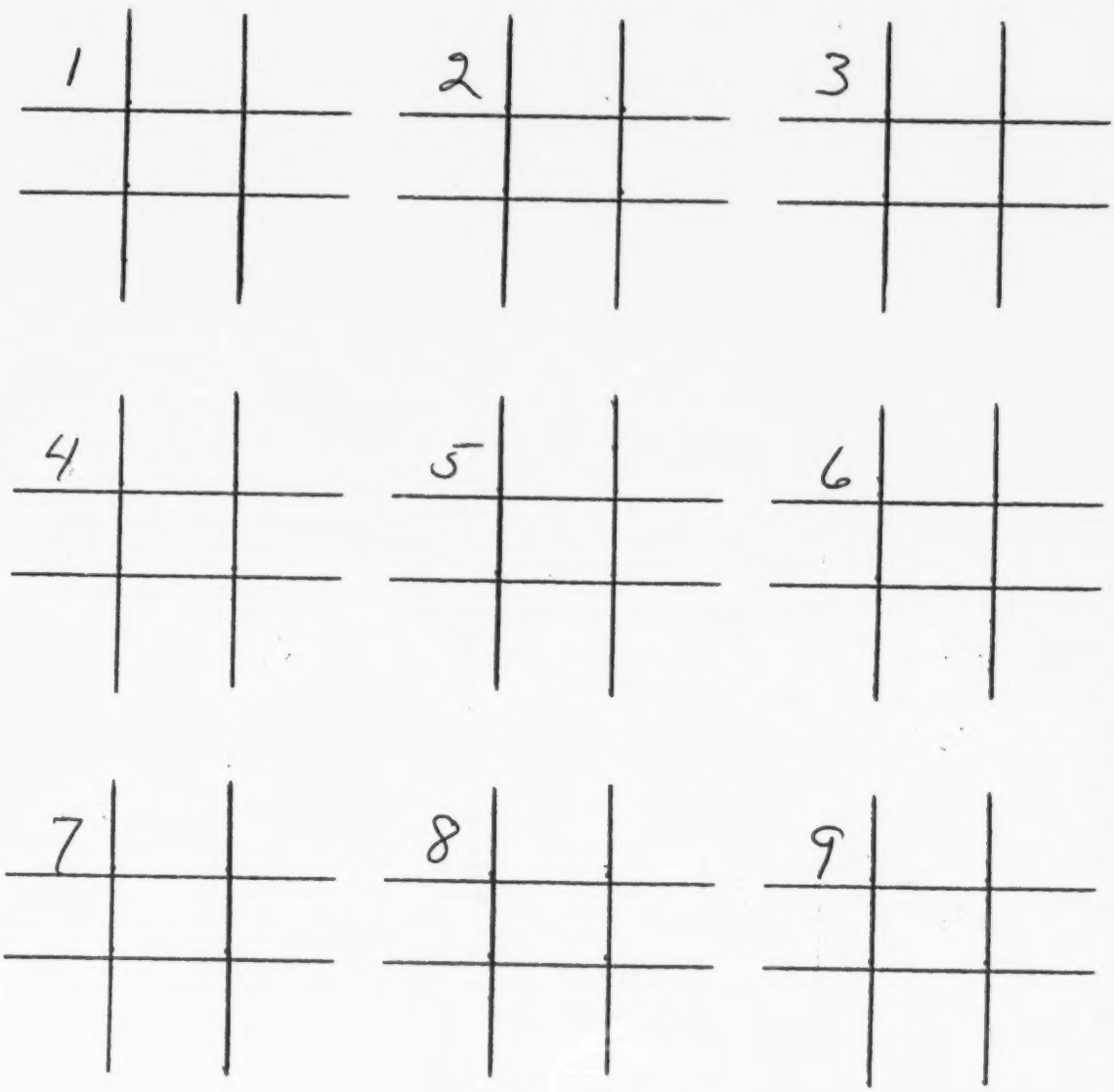
MORE PRACTICE

Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



MORE PRACTICE

Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



USING THE GRIDS

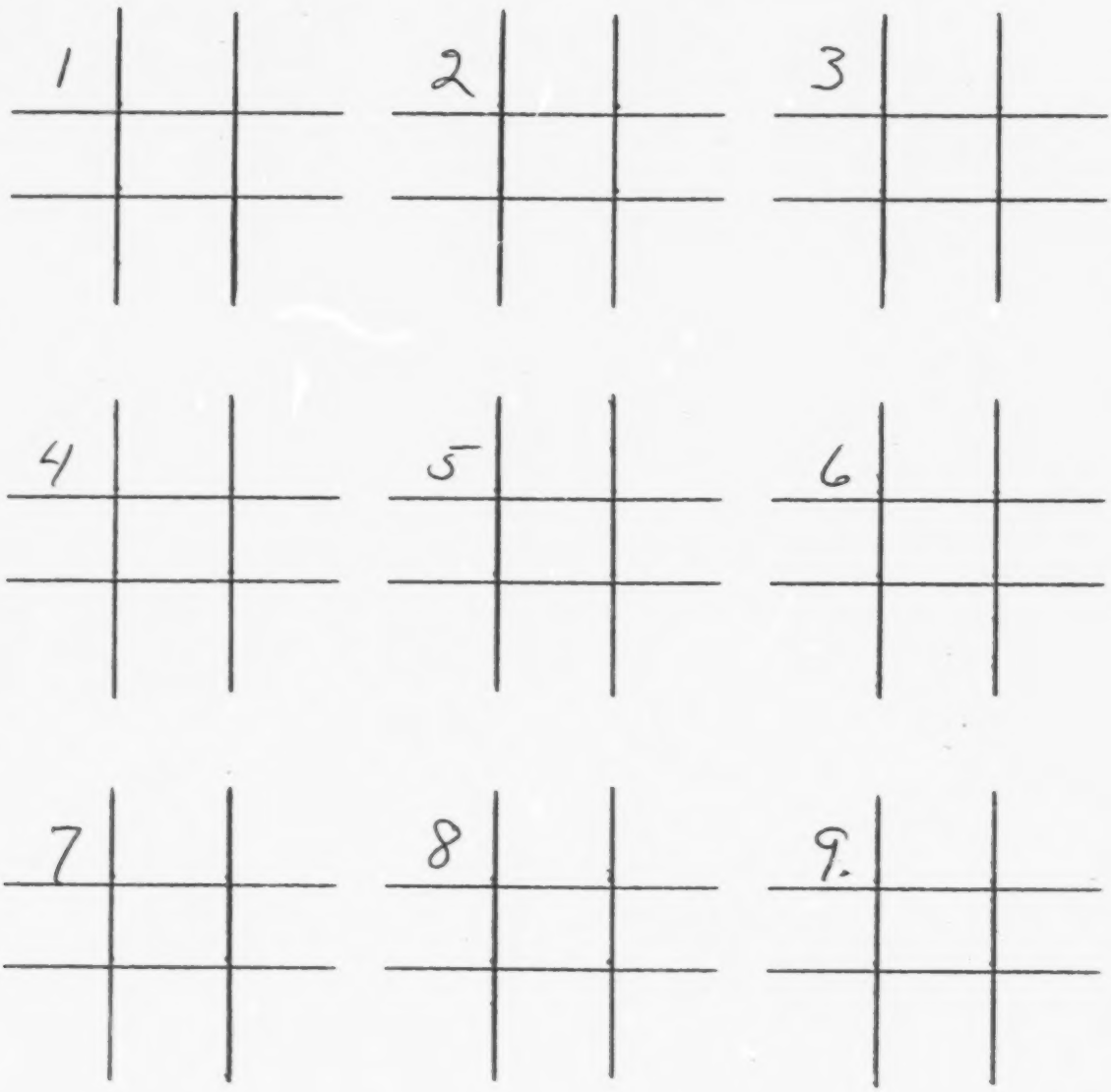
MULTIPLICATON

Now that you can produce the multiplication tables, you can solve problems easier than ever before. Refer to the grids when solving the following problems. If you become confused when doing a problem simply refer back to the grid to check your answers.

24 x5	63 x7	42 x2	74 x6	81 x9
57 x4	98 x3	16 x1	28 x9	33 x8
375 x9	489 x2	127 x4	543 x5	933 x7
410 x1	607 x5	315 x7	683 x6	744 x4
101 x3	855 x8	931 x2	246 x9	979 x5
3,021 x7	6,404 x6	9,005 x2	5,200 x3	3,301 x5
3,927 x4	6,534 x7	2,395 x1	7,310 x5	1,979 x9

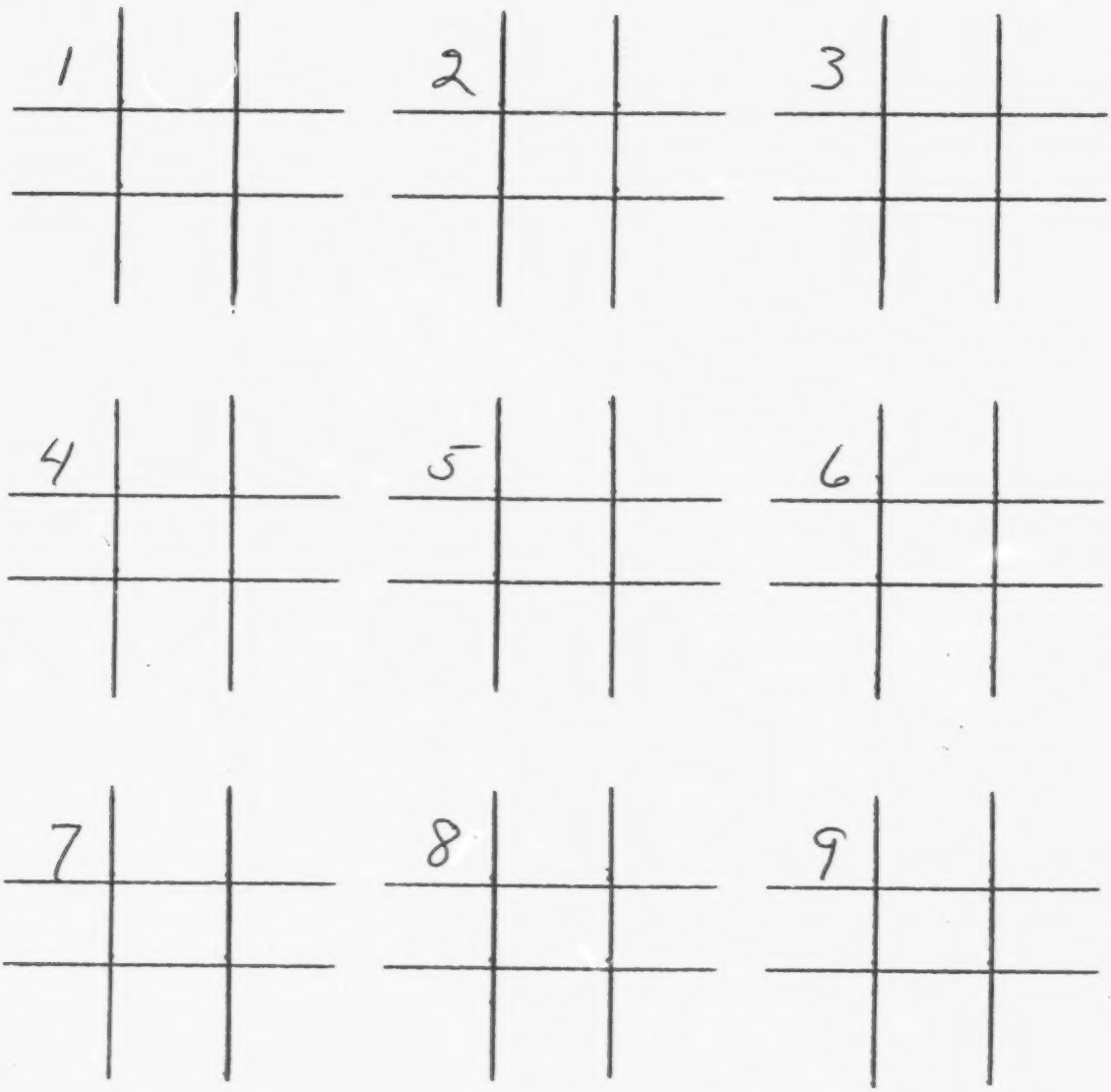
MORE PRACTICE

Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



MORE PRACTICE

Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



USING THE GRIDS

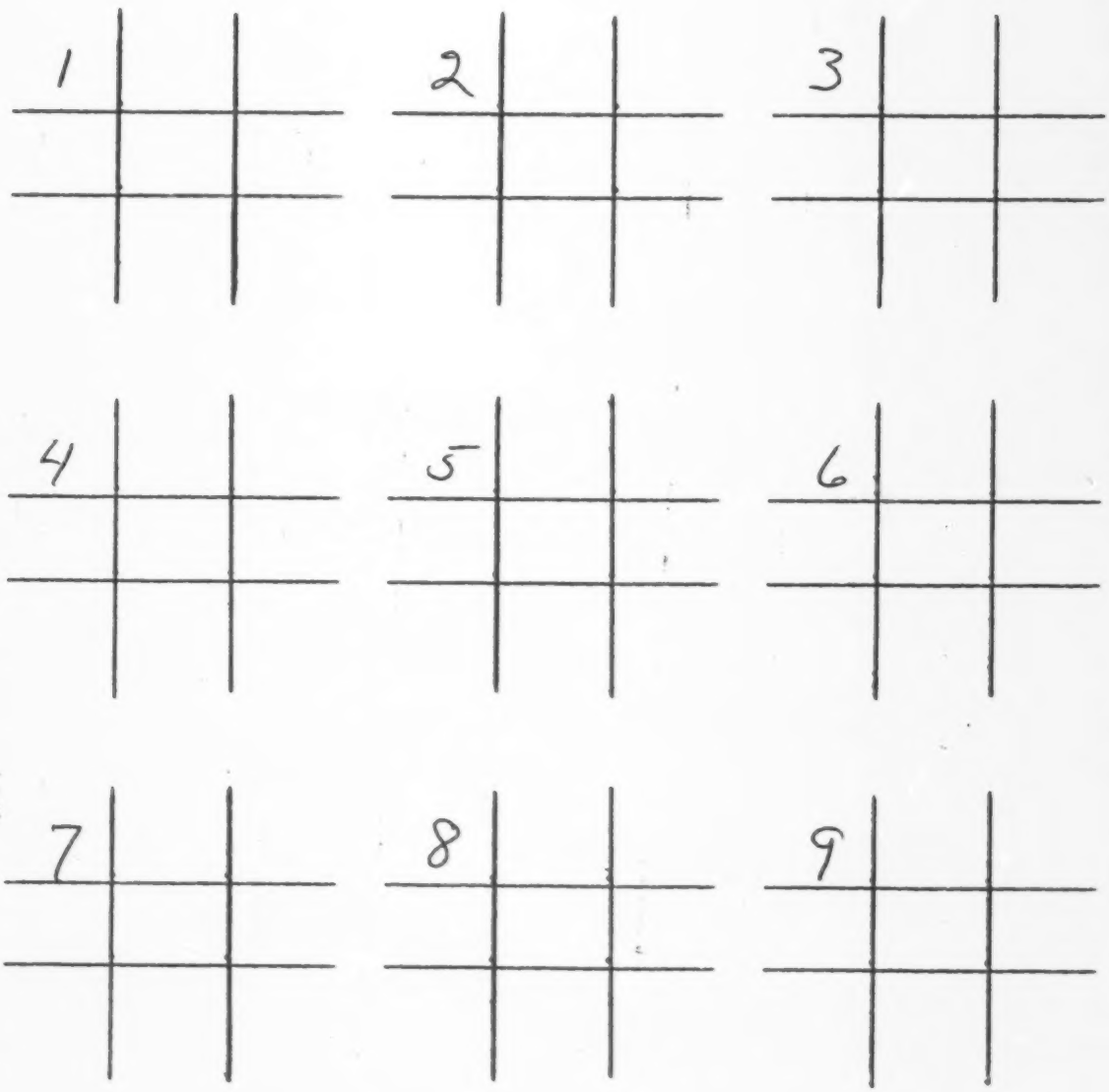
DIVISION

You can also use the multiplication grids to solve long division problems easier than ever before. Refer to the grids when solving the following problems. Find the number in the grid that comes closest without going over. Place the number of the position on top of the division problem. Copy the number that is in that position in the grid under the number in your division problem. Now substract and bring down the next number and repeat the procedure. If you have any difficulty consult with your instructor.

6) 546	3) 189	7) 497	4) 524
5) 255	2) 864	9) 729	3) 1596
8) 2554	6) 8642	5) 3195	7) 3543
9) 1064	2) 7348	4) 7264	3) 4472

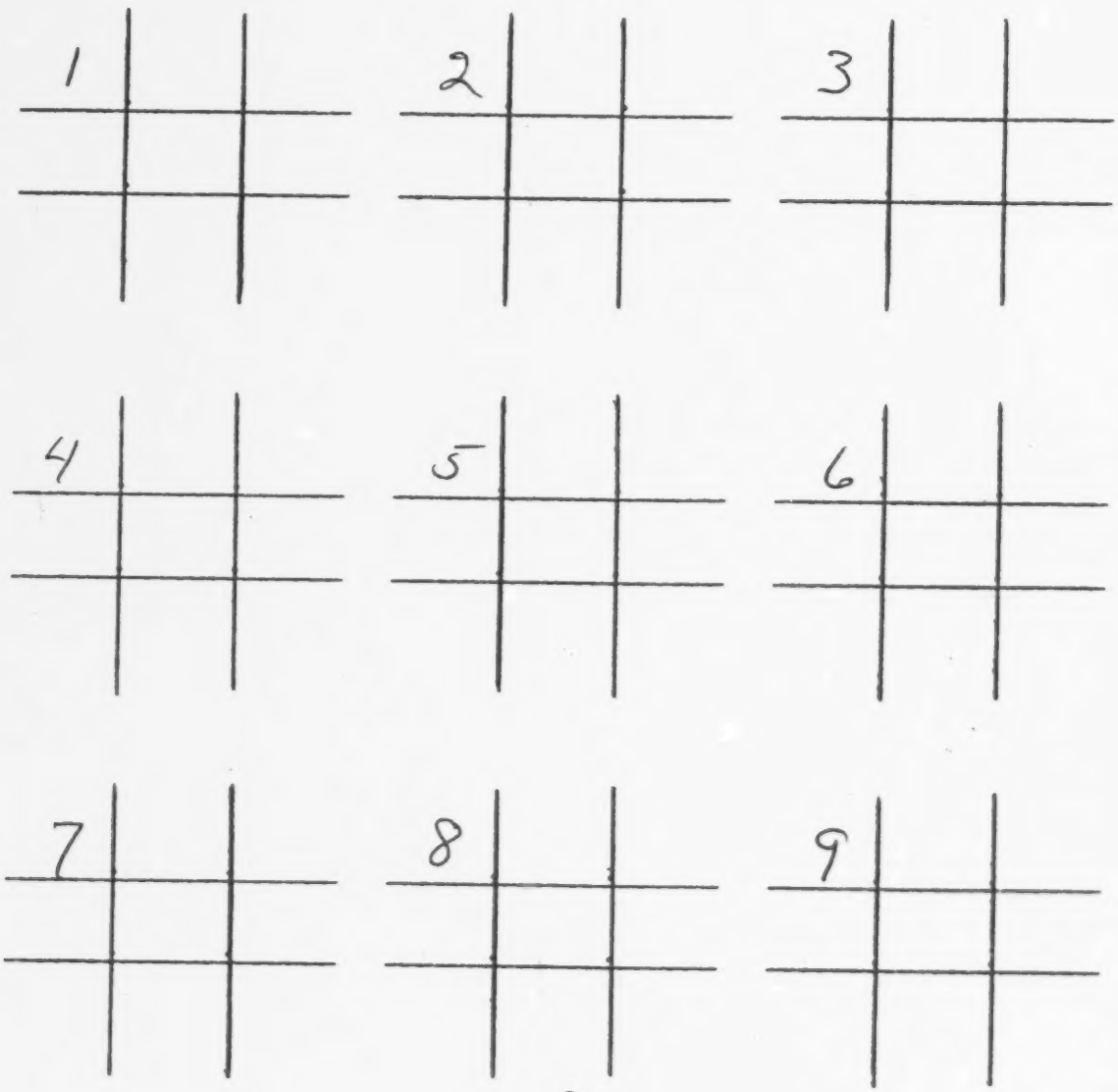
MORE PRACTICE

Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



MORE PRACTICE

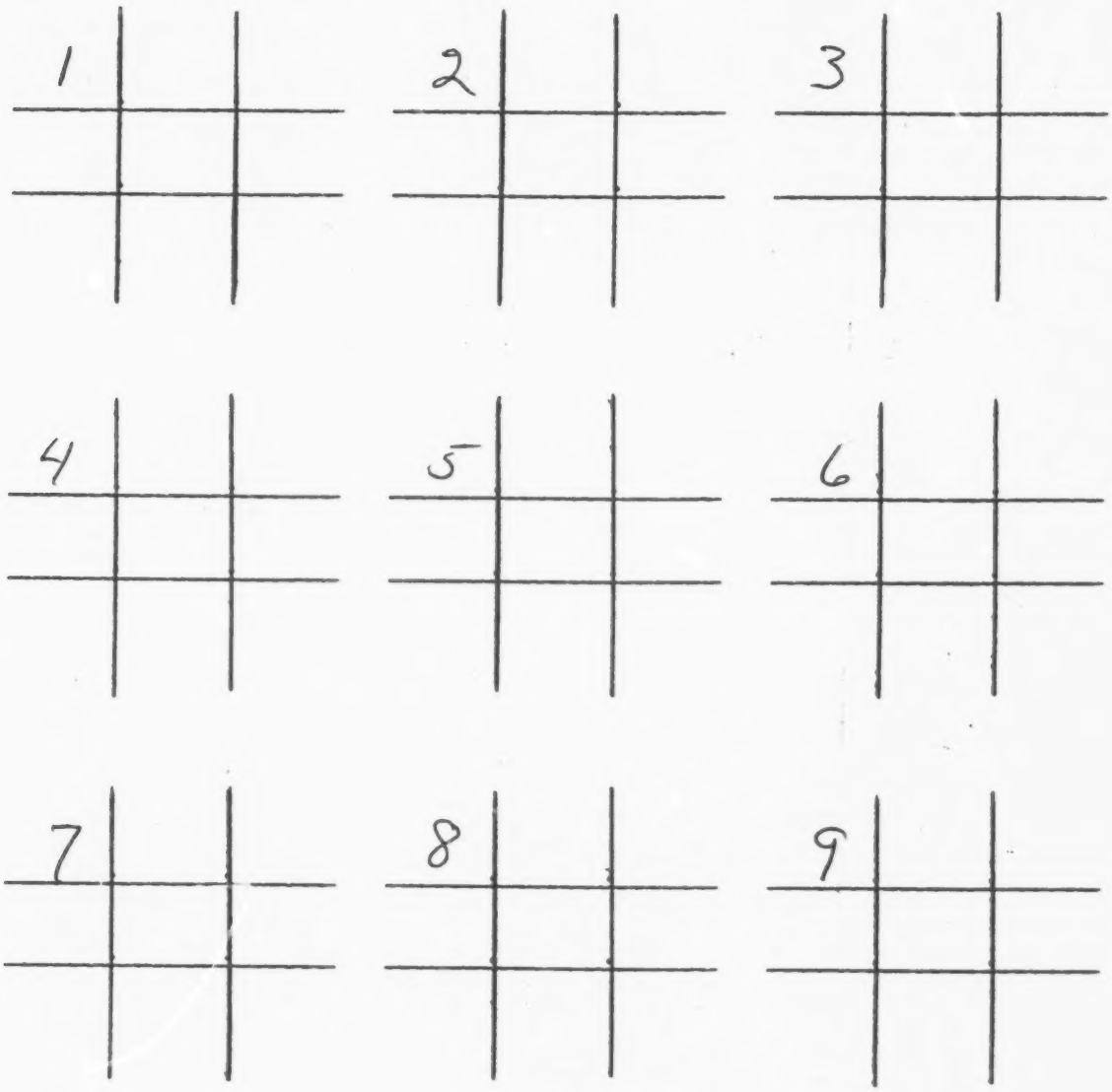
Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



87

MORE PRACTICE

Complete the grids each day for a couple of days until you can do them easily. If you need more practice draw the grids on scrap paper.



DOUBLE DIGIT GRIDS

Now that you have mastered the grids for the tables 1 through 9, it is time to begin double digit times tables. Below you have the example of the 10 times tables. It is formed simply by placing a 0 behind each number in the grid. Complete the rest of the grids and place a 0 behind each number.

10	40	70
20	50	80
30	60	90

20		

30		

40		

50		

60		

70		

80		

90		

DOUBLE DIGIT GRIDS
NUMBERS ENDING IN 1

The double digit grids which end with a 0 are very easy to form. So are those numbers which end with a 1, e.g. 21, 31, 41, etc. These grids are formed by placing the 1 times tables behind the completed grid. The first two are completed for you. Complete the rest.

11	44	77
22	55	88
33	66	99

21	84	147
42	105	168
63	126	189

31		

41		

51		

61		

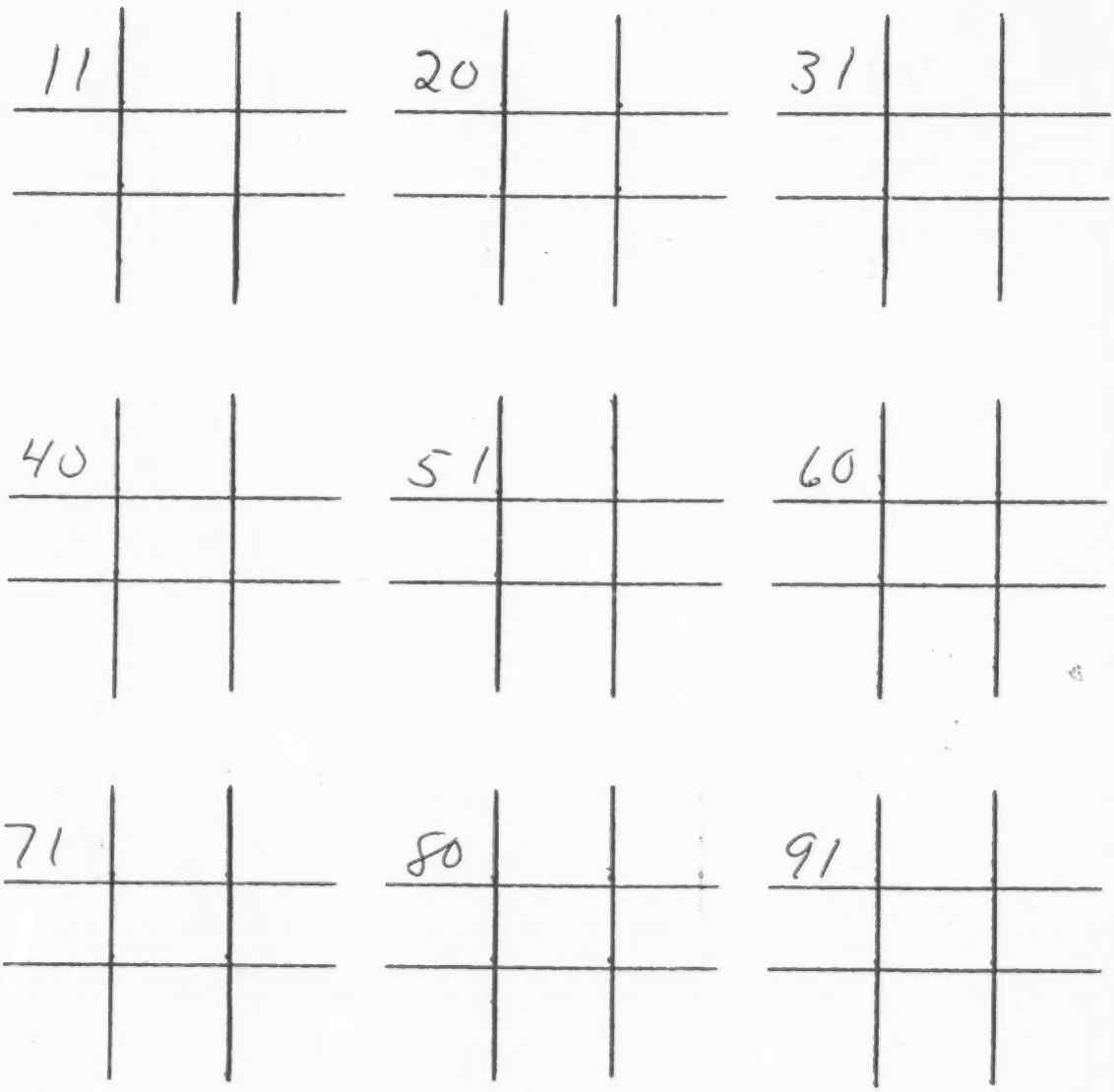
71		

81		

91		

DOUBLE DIGIT GRIDS
MORE PRACTICE

Complete the grids for the following numbers. If you need more practice, draw grids on scrap paper and complete the grids.



APPENDIX A

The grids with the numbers in the one's column completed.

1	4	7
2	5	8
3	6	9

2	8	4
4	0	6
6	2	8

3	2	1
6	5	4
9	8	7

4	6	8
8	0	2
2	4	6

5	0	5
0	5	0
5	0	5

6	4	2
2	0	8
8	6	4

7	8	9
4	5	6
1	2	3

8	2	6
6	0	4
4	8	2

9	6	3
8	5	2
7	4	1

APPENDIX B

The grids with the numbers in both the ten's and one's columns completed.

1	4	7
2	5	8
3	6	9

2	8	14
4	10	16
6	12	18

3	12	21
6	15	24
9	18	27

4	16	28
8	20	32
12	24	36

5	20	35
10	25	40
15	30	45

6	24	42
12	30	48
18	36	54

7	28	49
14	35	56
21	42	63

8	32	56
16	40	64
24	48	72

9	36	63
18	45	72
27	54	81

The answers to the multiplication problems

24 x5 = 120	63 x7 = 441	42 x2 = 84	74 x6 = 444	81 x9 = 729
57 x4 = 228	98 x3 = 294	16 x1 = 16	28 x9 = 252	33 x8 = 264
375 x9 = 3375	489 x2 = 978	127 x4 = 508	543 x5 = 2,715	933 x7 = 6,531
410 x1 = 410	607 x5 = 3,035	315 x7 = 2205	683 x6 = 4,098	744 x4 = 2,976
101 x3 = 303	855 x8 = 6,840	931 x2 = 1,862	246 x9 = 2,214	979 x5 = 4,895
3,021 x7 = 21,147	6,404 x6 = 38,424	9,005 x2 = 18,010	5,200 x3 = 15,600	3,301 x5 = 16,505
3,927 x4 = 15,708	6,534 x7 = 45,738	2,395 x1 = 2,395	7,310 x5 = 36,550	1,979 x9 = 17,811

The answers to the division problems

$\begin{array}{r} 91 \\ 6\overline{)546} \end{array}$ $\begin{array}{r} 63 \\ 3\overline{)189} \end{array}$ $\begin{array}{r} 71 \\ 7\overline{)497} \end{array}$ $\begin{array}{r} 131 \\ 4\overline{)524} \end{array}$

$\begin{array}{r} 51 \\ 5\overline{)255} \end{array}$ $\begin{array}{r} 432 \\ 2\overline{)864} \end{array}$ $\begin{array}{r} 81 \\ 9\overline{)729} \end{array}$ $\begin{array}{r} 532 \\ 3\overline{)1596} \end{array}$

$\begin{array}{r} 317\ R2 \\ 8\overline{)2554} \end{array}$ $\begin{array}{r} 1440\ R2 \\ 6\overline{)8642} \end{array}$ $\begin{array}{r} 639 \\ 5\overline{)3195} \end{array}$ $\begin{array}{r} 506\ R1 \\ 7\overline{)3543} \end{array}$

$\begin{array}{r} 118\ R4 \\ 9\overline{)1064} \end{array}$ $\begin{array}{r} 3674 \\ 2\overline{)7348} \end{array}$ $\begin{array}{r} 1816 \\ 4\overline{)7264} \end{array}$ $\begin{array}{r} 1490\ R2 \\ 3\overline{)4472} \end{array}$

Tic Tac Toe Math

Workbook II

Double and Multi-digit

By

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Published by:

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Introduction

Workbook II is designed to develop the student's ability to create and use multiple digit Tic Tac Toe grids. The student should have already completed Work Book I which helps the students learn to make the single digit grids.

Workbook I taught the student of Tic Tac Toe how to identify the positions 1 to 9 in the grids and how to use the pattern on the following page to create the grids 1 through 9. Workbook I also introduced the student to the double diget grids ending in 0 and 1 (10-11 - 20-21, etc.).

Workbook II will take the student through the process of creating and using double digit grids (other than those ending in 0 and 1) and multiple digit grids. This workbook shows how to use the grids to make long division a breeze.

Tic Tac Toe Patterns

Pattern 1 There is always a five or a zero in the center position.

Pattern 2 All opposite positions on the x and the cross must add to ten.

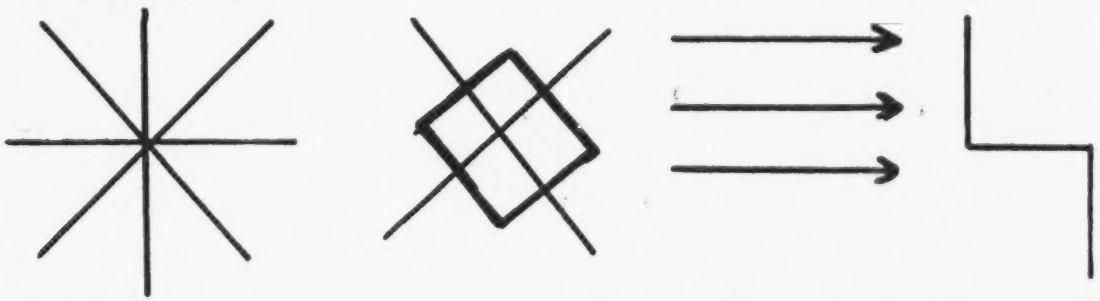
Pattern 3 The second position is obtained by doubling the number in the first position.

Pattern 4 Opposite positions on the X and the cross add up to ten times the first number when the grids are completed.

Pattern 5 Grids of odd numbers always have odd numbers on the X and even numbers on the diamond.

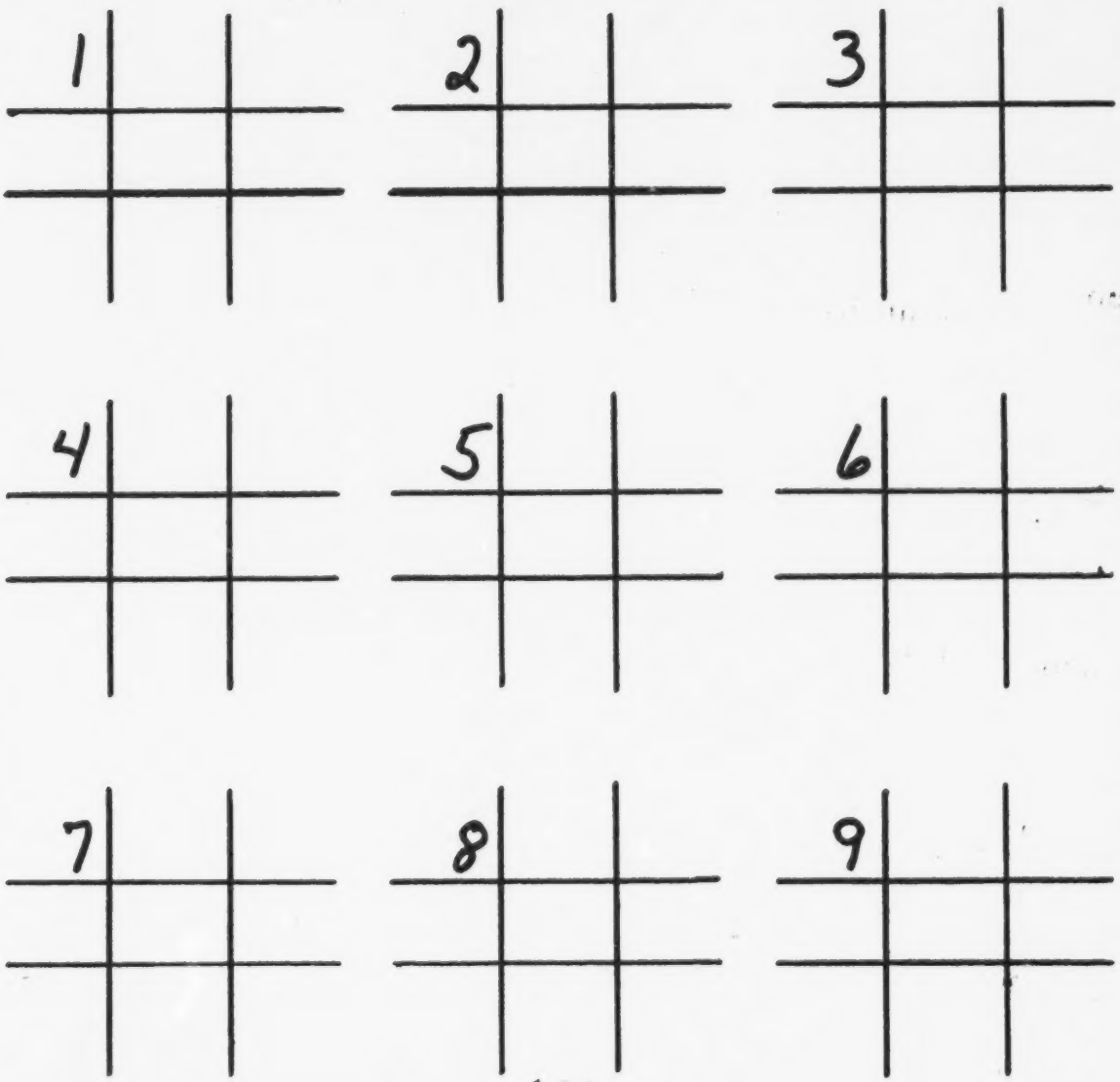
Pattern 6 There are no odd numbers in the even grids.

Visual Pattern



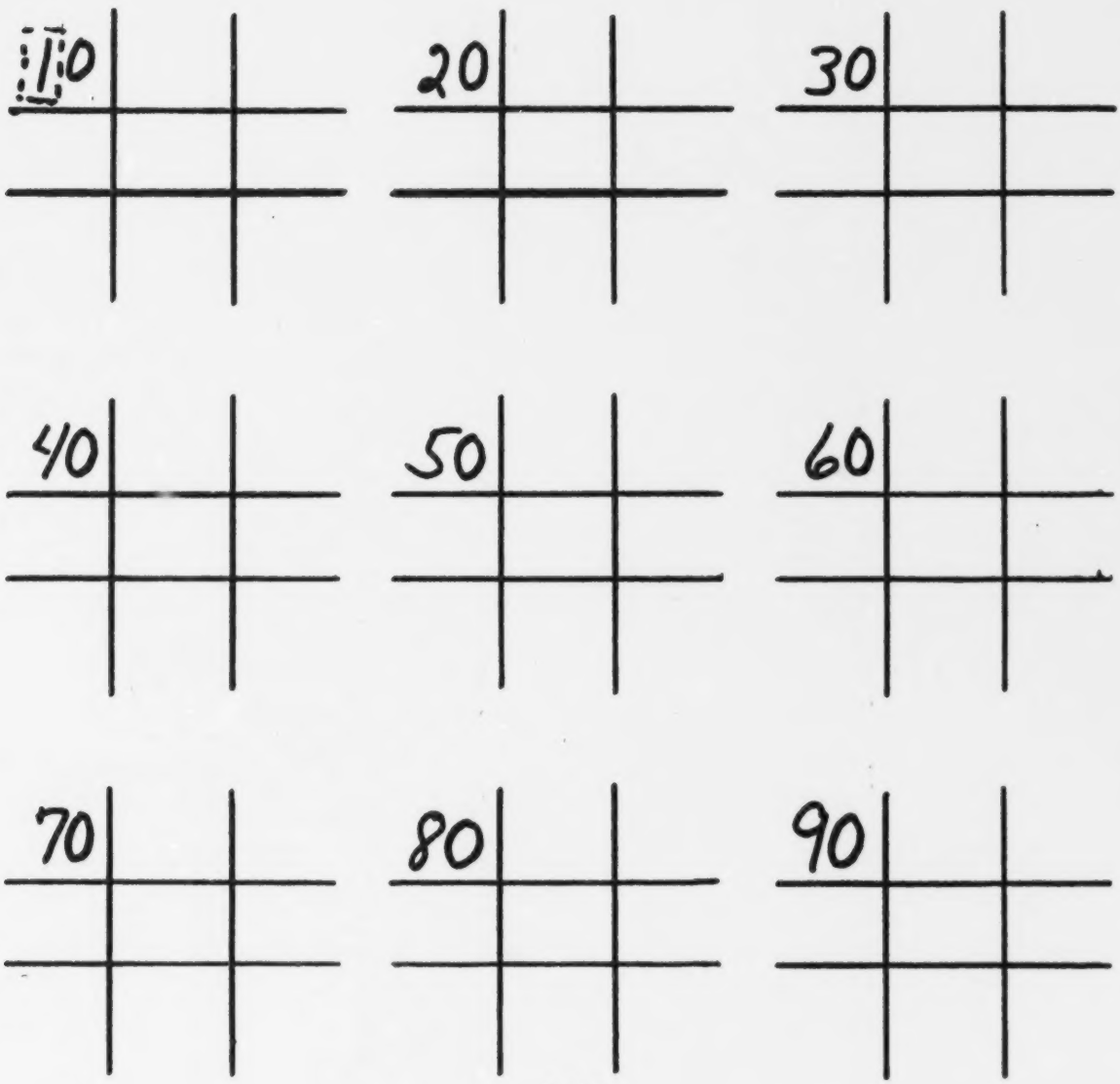
Tic Tac Toe Math Review

Complete the following grids: (If you do not know how or have forgotten any part of them, consult the Tic Tac Toe Math Instructional Guide or Workbook I to learn the system or refresh your memory.)



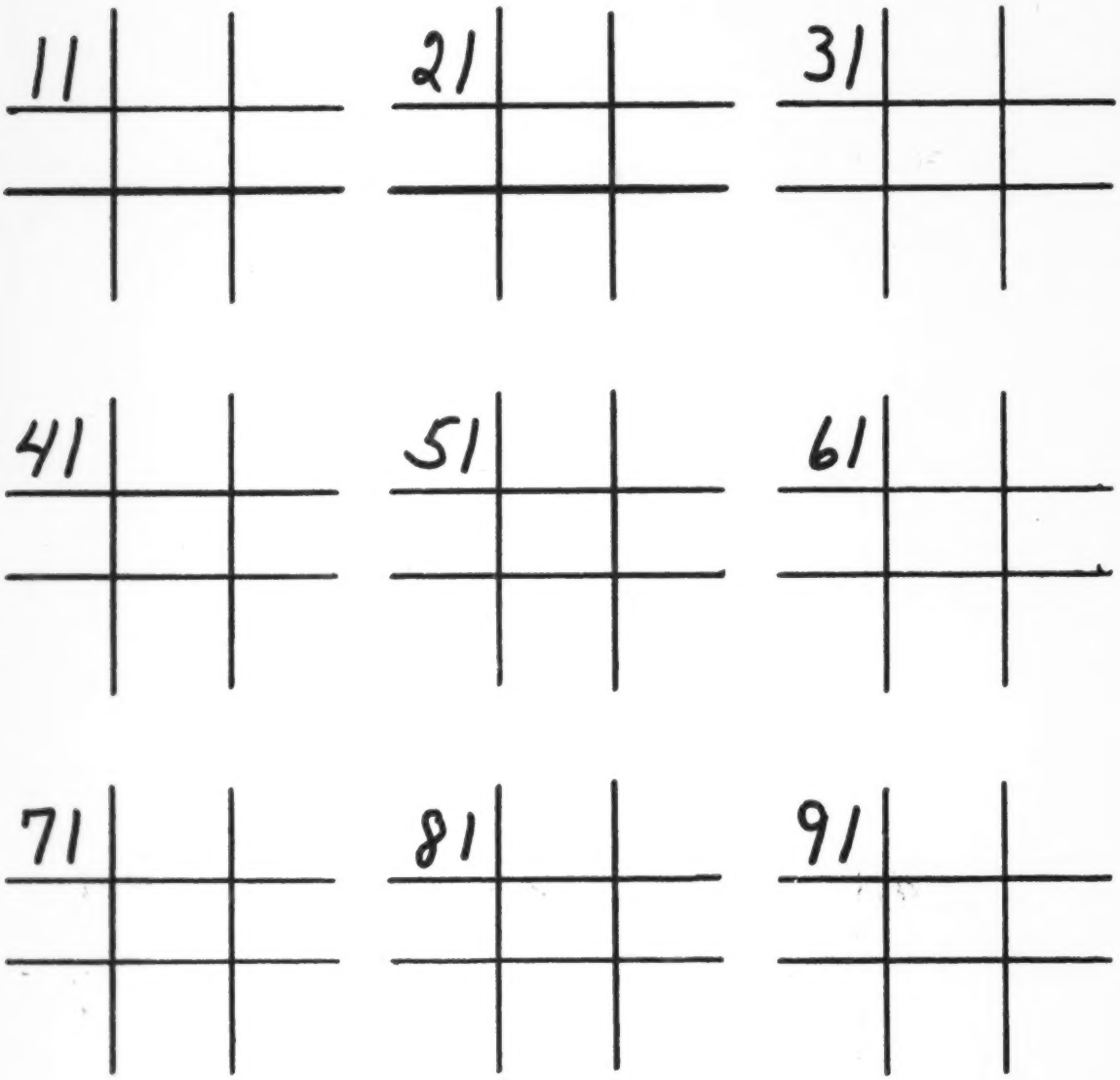
Review Double Digit Grids Ending with 0.

In Workbook I, you learned to create the grids for 10, 20, 30, 40, 50, 60, 70, 80, and 90. Take a moment and review the grids. (Remember, first complete the single digit grids and then add the zeroes.) Note: So that you will not have to squeeze in your numbers, be sure you place the numbers for the grids (1 to 9) in the middle of each position.



Review: Double Digit Tic Tac Toe Grids Ending in 1.

In Workbook I, you also learned how to complete the grids for 11, 21, 31, 41, 51, 61, 71, 81, and 91. Complete the following grids. (Remember to do the single digit grid first, placing the number in the middle of the grid and then add the one-times table behind the completed single digit grids.)



Review: Using the Single digit Grids

Complete the following exercise.

8×3	5×2	9×4	1×6	7×7	3×8
6×5	9×1	2×7	5×8	9×6	3×4
12×8	25×5	47×9	38×4		

$9\overline{)81}$ $5\overline{)35}$ $6\overline{)24}$ $3\overline{)18}$

$8\overline{)648}$ $2\overline{)2468}$ $4\overline{)4914}$

Review: Using Double Digits Ending With 0.

Complete the following exercise.

6×10	9×20	7×40	5×50
10×10	30×60	50×90	70×20
$70\overline{)560}$	$20\overline{)120}$	$50\overline{)350}$	
$30\overline{)240}$	$10\overline{)800}$	$90\overline{)270}$	
$40\overline{)280}$	$60\overline{)300}$	$80\overline{)640}$	

Review: Using Double Digits Ending in 1.

Complete the following exercise.

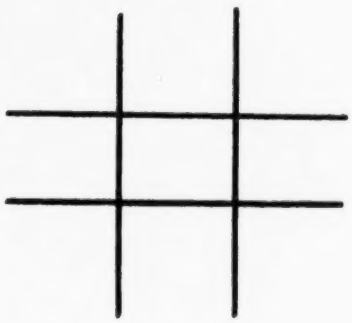
6 x 31	4 x 81	9 x 11	1 x 51
8 x 71	3 x 41	5 x 61	4 x 11

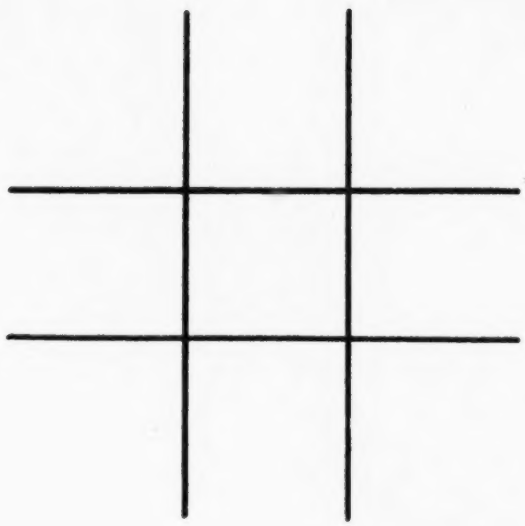
$51\overline{)357}$	$81\overline{)405}$	$21\overline{)168}$
$41\overline{)123}$	$11\overline{)7777}$	$71\overline{)284}$
$61\overline{)549}$	$91\overline{)366}$	$31\overline{)248}$

Double Digit Numbers Ending With 2, 3, 4, 5, 6, 7, 8, and 9

Once you can createt single digit grids with ease and can complete double digit numbers with 0 and 1, you are ready for the rest of the double digits. These are very easy to create and make solving double digit long division more like a game than a problem.

The double digit grids used in numbers 2, 3, 4, 5, 6, 7, 8 and 9 need 2 grids, one of regular size and a second which is a bit larger.



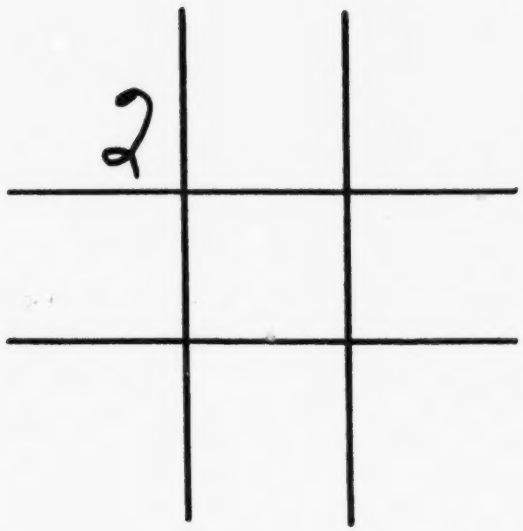


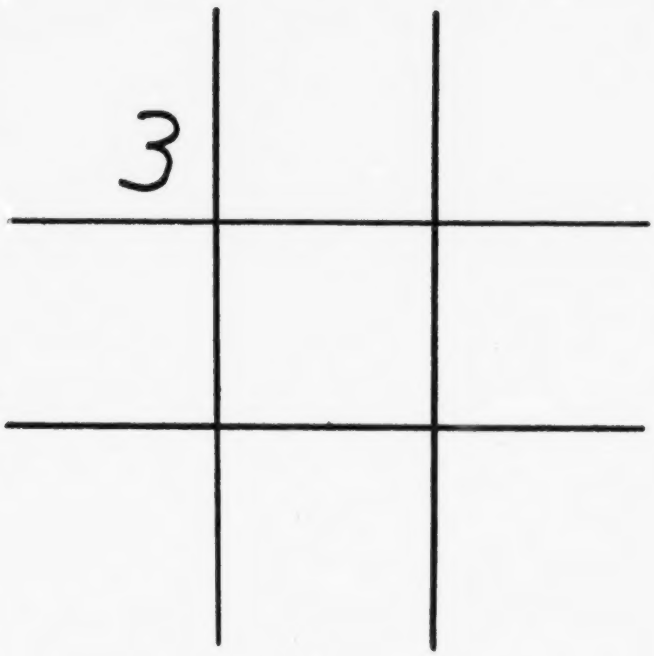
Creating Double Digit Grids

In each of the grids you will comlplete a Tic Tac Toe for one of the two numbers. For example, to form the Tic Tac Toe grid for 23:

First: Make two grids

Second: Place the 2 in the regular grid and the 3 in the second grid. Your grids will look like this.

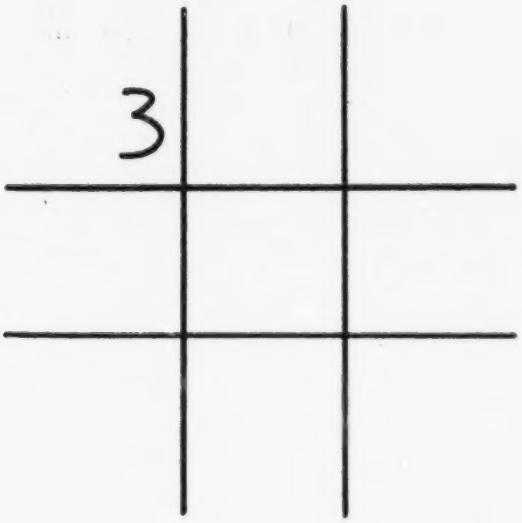




Turn the page for the next step.

The third step is to complete the grid (2) on the left.

2	8	14
4	10	16
6	12	18

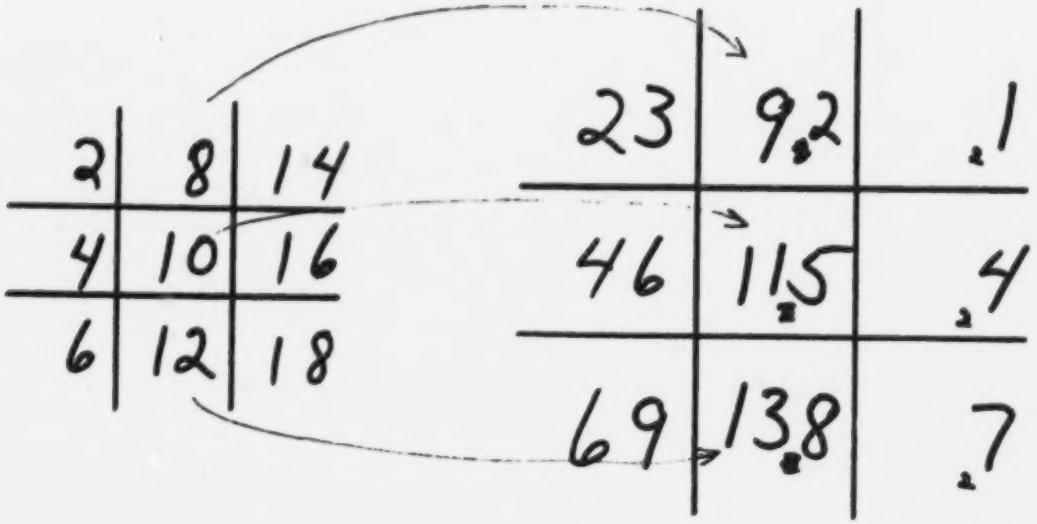


The fourth step is to complete the pattern (single digits, one's column only) in the grid (3) on the left. Your grids now look like this.

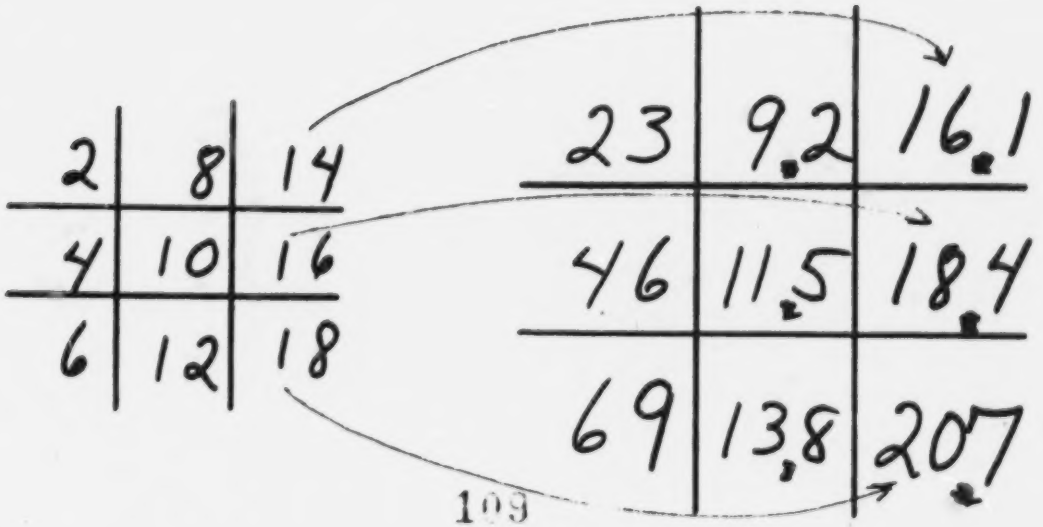
2	8	14
4	10	16
6	12	18

3	2	1
6	5	4
9	8	7

Now bring the number in the fourth position to the fourth position in the right grid and add it to the small number which you find there. The 1 is added to the 8, and you write a 9. Erase or scratch out the little number so that you don't get confused later.



Repeat this process for the rest of the grid.

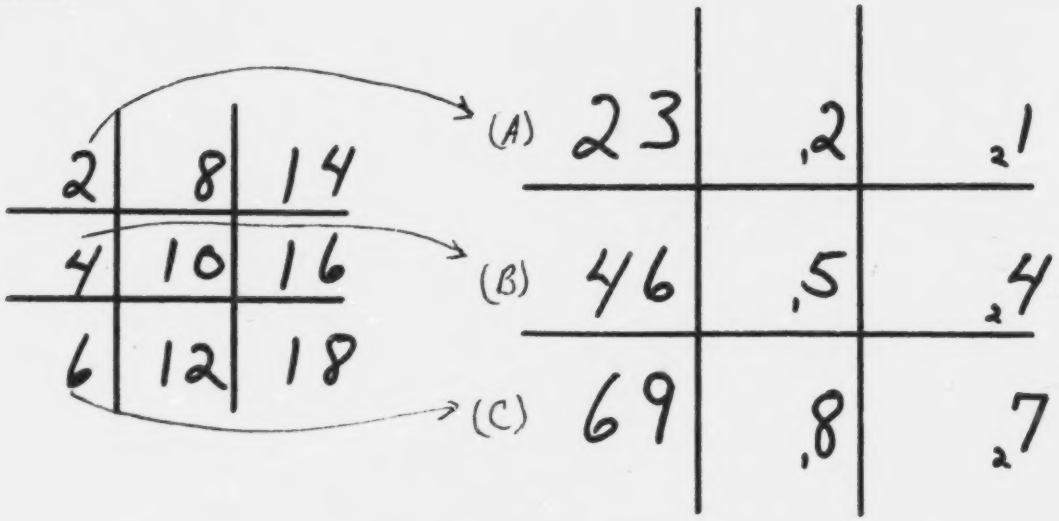


The fifth step is to use small numers to complete the grid on the right (3). Your grids will now look like this.

2	8	14
4	10	16
6	12	18

3	.2	$_2$1
6	.5	$_2$4
9	.8	$_2$7

The sixth step in the process is to combine the grids to produce the 23 times table. Bring the number (2) in the first position to the first position in the grid on the right (A), giving you 23.

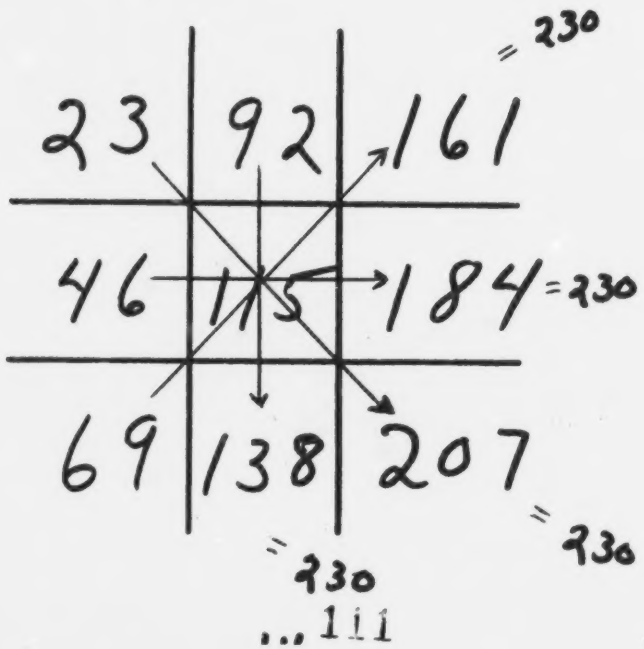


(B) Place the number in the second position (4) in the second position in the grid on the right. (C) Move the number from the third position (6) to the third position in the grid on the right.

When you have completed the process, you will have the 23-times table, which will look like this.

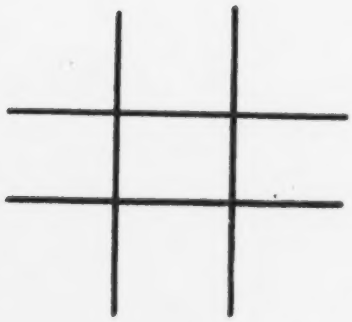
23	92	161
46	115	184
69	138	207

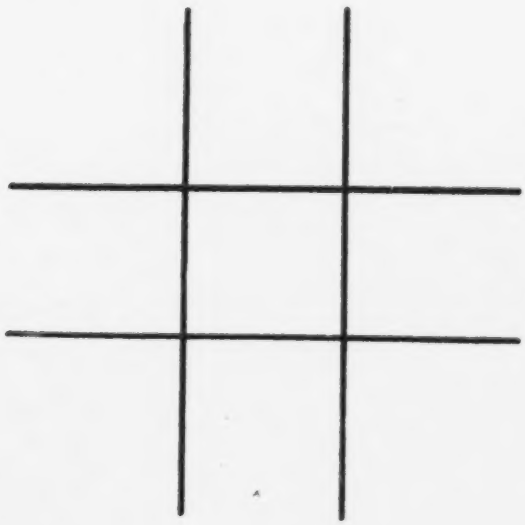
To check your grid, you can add opposite positions. The sum will be 10 times the first number (230). Each of the opposite positions in the grids always add up to 230. The number in the fifth position will always be 1/2 ten times the first number (115).



Let's try another, using the six steps for the 54 times table.

1) The first step is providing two grids.
2) Place the 5 in the left grid, the 4 in the right
3) Complete the 5 times table.
4) Complete the single digits of the 4 times table.
5) Complete the 4 times table with small numbers.
6) Combine the two grids.



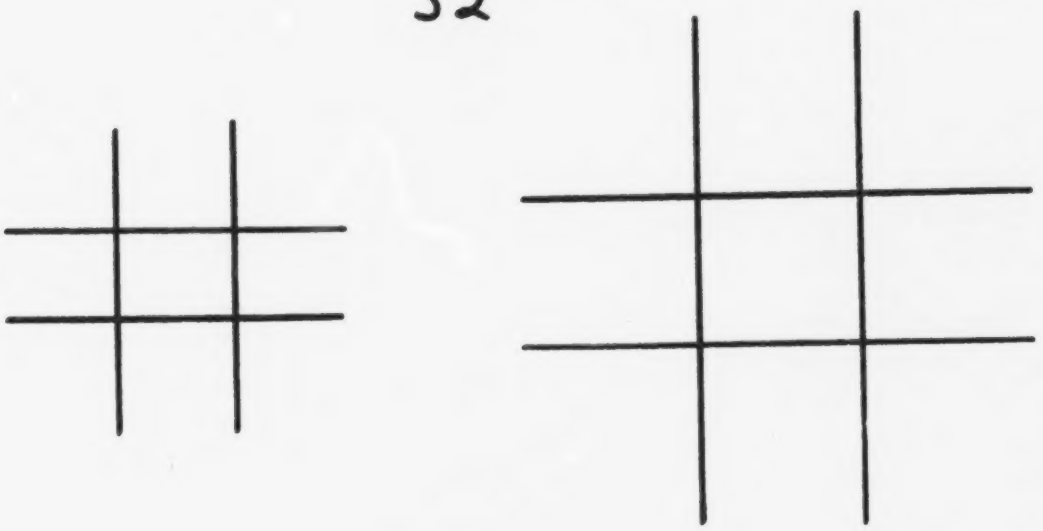


Check all opposite positions to see if they add to ten times the first number.

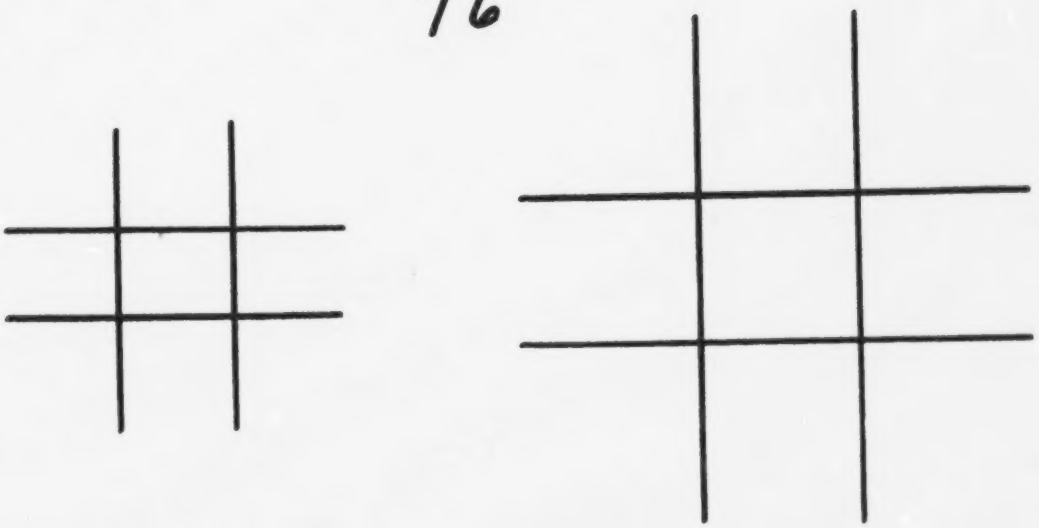
112

Now do the following double digit grids.

32

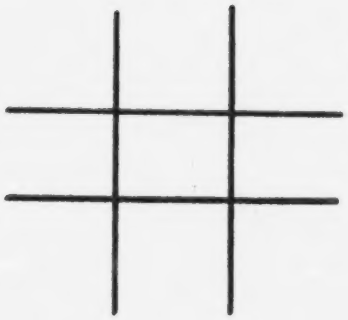


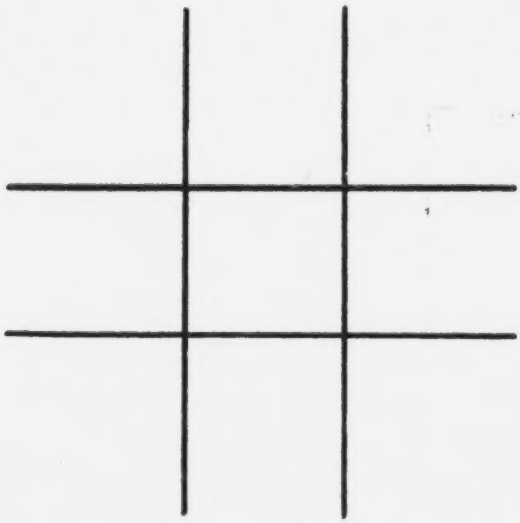
76



Check all opposite positions to see if they add to ten times the first number.

Complete the following grids, and do the problems.





$24\overline{)768}$

$24\overline{)19248}$

As you see, double digit grids are most useful for solving long division. When you make the grids, you will be able to solve long division problems as fast or faster than others who do them in traditional ways. You can use double digit for multiplication or the single digits and multiply as others would.

To solve multiplicaton problems with the double digit grids, you copy the number that is in the appropriate positions and add zeros to hold the place value as you would when multiplying the traditional way. Study the following problem.

1. Find the number in the 9th position of the 76 grid
2. Add a zero to hold the place value and place the number from the first position in front of it.
3. Add two zeros to hold the place values and place the number from the second position on the third line in front of the zeros.

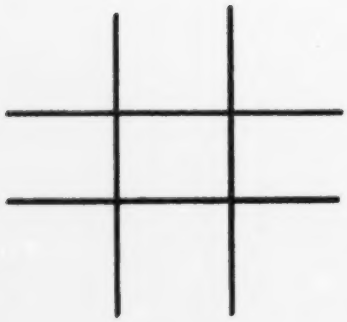
$$\begin{array}{r} 219 \\ \times\ 76 \\ \hline 684 \\ 760 \\ +15200 \\ \hline 16644 \end{array}$$

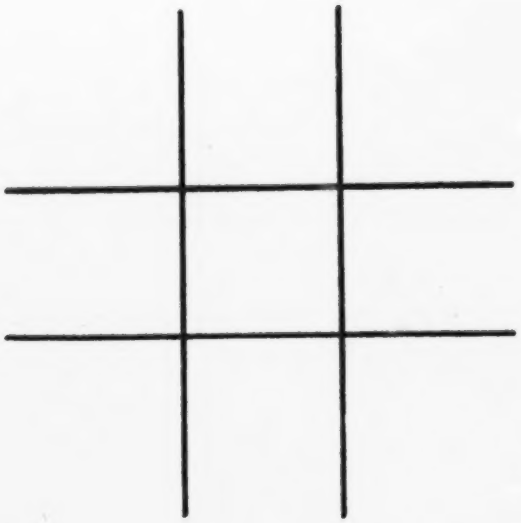
If you have difficulty with the zeros which hold the place values, set up the problem in the following way:

$$\begin{array}{r} 219 \\ \times\ 76 \\ \hline \\ \\ \\ \\ \hline \end{array}$$

Do the following grids and solve the problems.

48





$$\begin{array}{r} 63 \\ \times\ 48 \\ \hline \end{array}$$

$$\begin{array}{r} 19 \\ \times\ 48 \\ \hline \end{array}$$

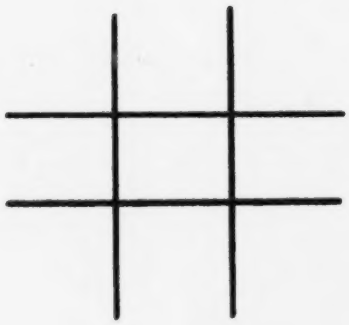
$$\begin{array}{r} 57 \\ \times\ 48 \\ \hline \end{array}$$

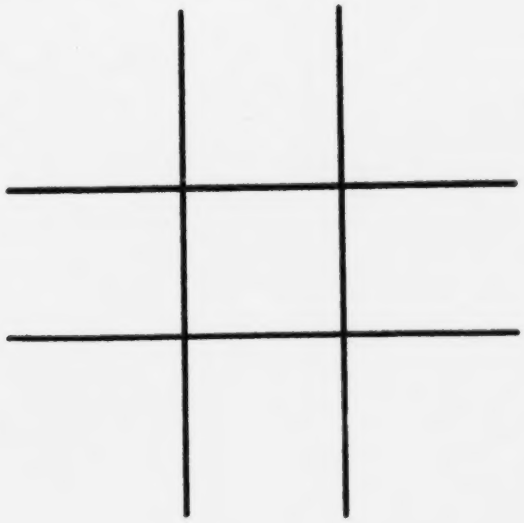
$$\begin{array}{r} 369 \\ \times\ 48 \\ \hline \end{array}$$

$$\begin{array}{r} 258 \\ \times\ 48 \\ \hline \end{array}$$

Complete the following grids and solve the problems.

86





$86\overline{)476}$

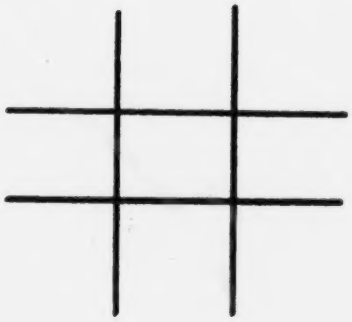
$86\overline{)16342}$

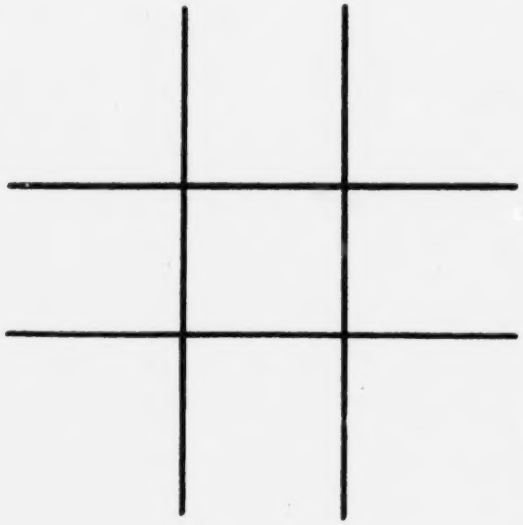
$86\overline{)56792}$

$86\overline{)33439}$

20

Complete the following grids and solve the problems.





$92\overline{)742836}$

$92\overline{)158946}$

Triple Digit Numbers Ending in 0

You will remember how easy it was to create the grids for 10, 11, 20, 21, and so on. Now you will see that it is as easy to create the grids for 320, 321, 450, 451, 840, 841, and so on.

Here is a completed grid for the 23 times table. To obtain the 230 table, simply place a 0 behind each number.

230	92_	161_
46_	115_	184_
69_	138_	207_

To obtain the 231 table, simply place the 1 times table behind each number.

231	92_	161_
46_	115_	184_
69_	138_	207_

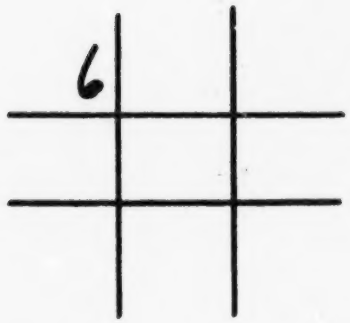
Now let's use these grids on a few problems.

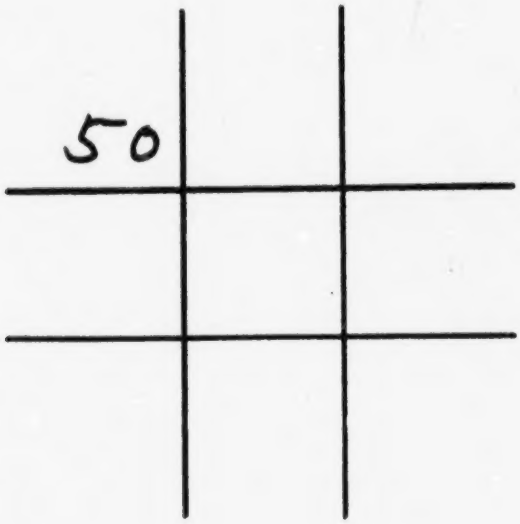
$230\overline{)4656}$ $230\overline{)9367}$

$231\overline{)136482}$ $231\overline{)5489377}$

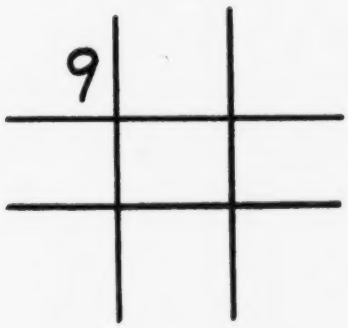
Try some yourself. Remember to leave room for the zeroes and the small number in each position.

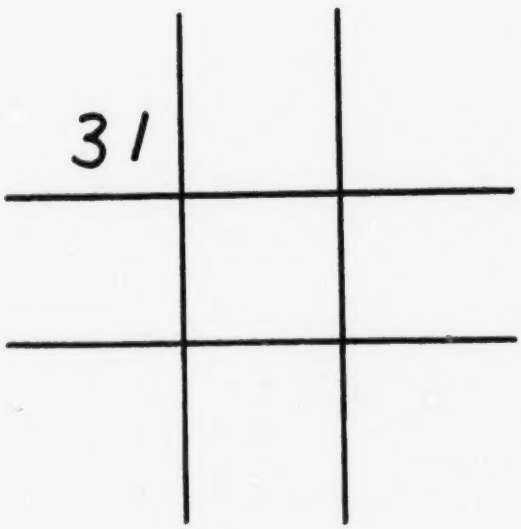
650





931





To form multi-digit grids which do not contain zeros or ones, you need the same number of grids as there are digits. Each additional grid needs to be large enough to accommodate the larger numbers. Here is an example of a 3-digit grid.

When you move 23 to position one to form 234, 46 goes to position 2 to form 468. 69 moves to position 3, and you add the small number to 69 resulting in 702 in third position and so on.

234

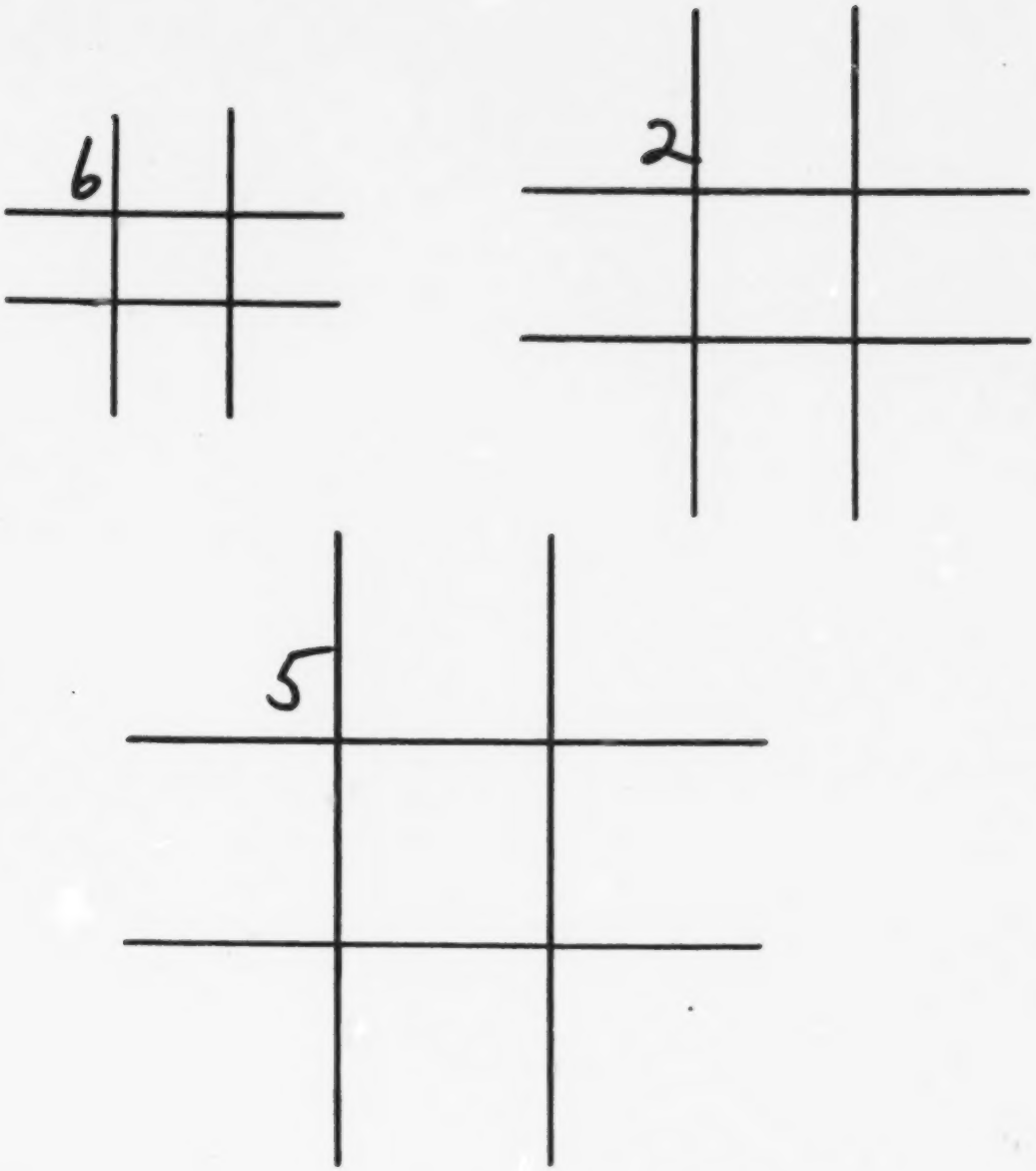
2	8	14
4	10	16
6	12	18

23	$9_{,2}$	$16_{,2}1$
46	11,5	$18_{,2}4$
69	$13_{,}8$	$20_{,2}7$

234	$93_{,}6$	$163_{,2}8$
468	$117_{,2}0$	$187_{,3}2$
$70_{,}2$	$1404_{,2}$	$210_{,3}6$

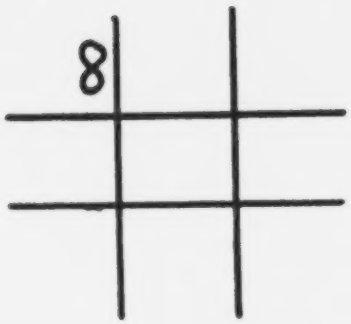
Your turn.

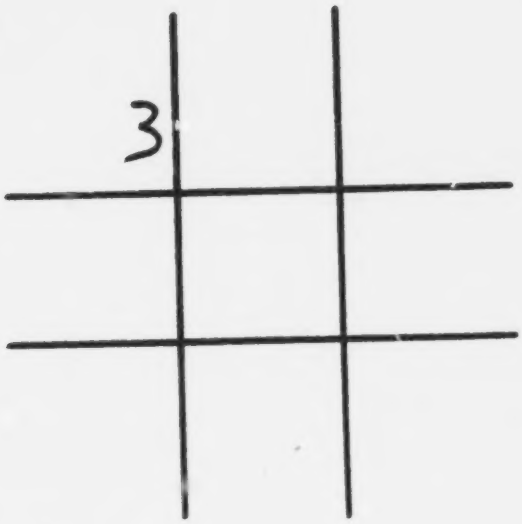
625

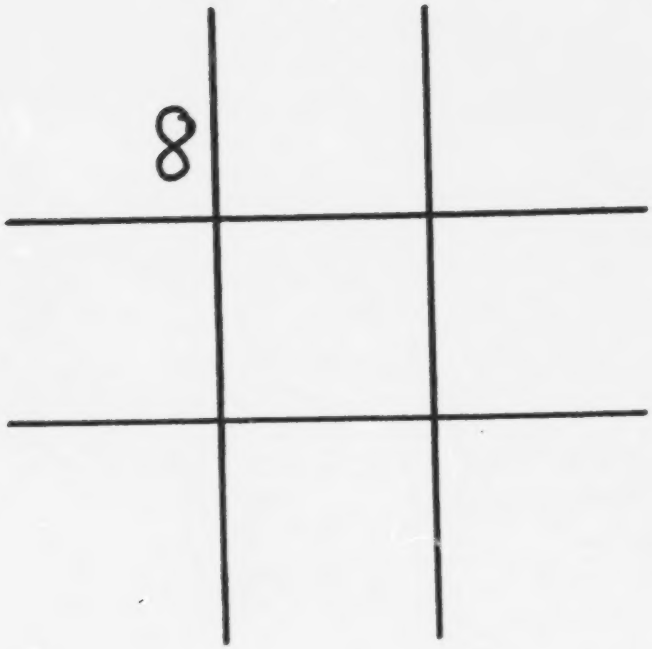


Complete the grids, and use it to solve this problem.

838







838)46846 52

838)18943198

Here is a sample of a four-digit table. 3,672

3	12	21
6	15	24
9	18	27

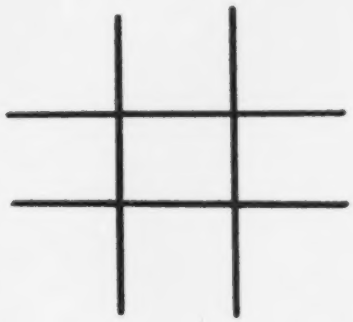
36	144	252
72	180	288
108	216	324

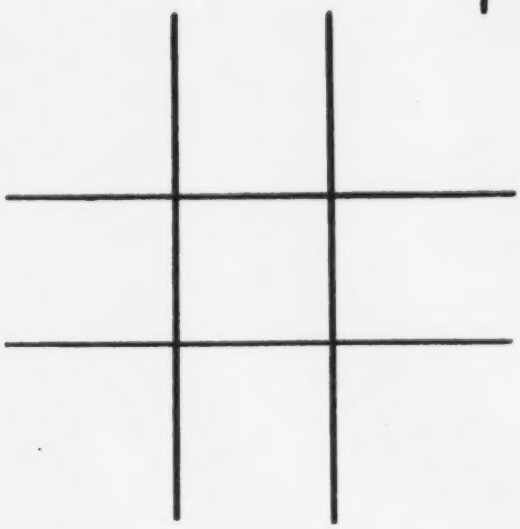
367	1468	2569
734	1835	2936
1101	2202	3303

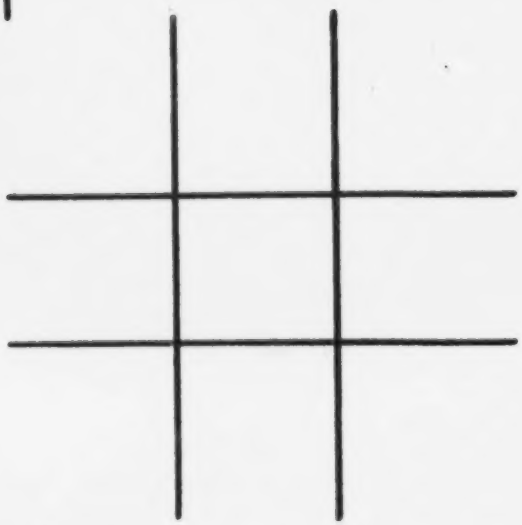
3672	14688	25704
7344	18360	29376
11016	22032	33048

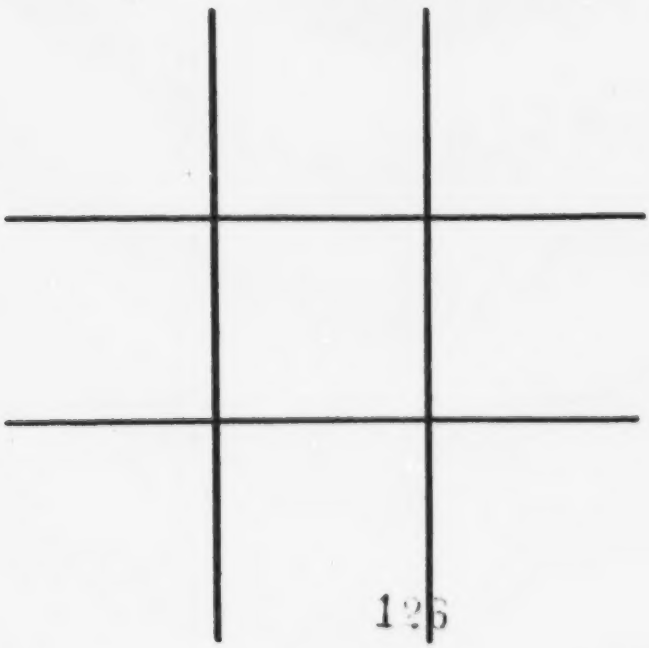
Try the grids on this page, then make a problem and solve it.

4,796









TIC TAC TOE MATH

WORKBOOK III

FRACTIONS

BY

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Tic Tac Toe Math Used with Fractions

Tic Tac Toe Math can help students improve their abilitiy to do fractions. Many students have not been able to learn how to do calculations with fractions because manyof the procedures require knowledge of the times tables. Students who did not know the times tables had great difficulty reducing fractions, finding common denominators and changing mixed and improper fractions. Tic Tac Toe Math provides the students with the opportunity to learn the procedures and become more proficient with fractions.

Avoidance, whether caused by lack of knowledge or dislike for a subject, results in limited time on task. And time on task without understanding is the same as no time on task. The only way to learn how to do calculations with fractions is to do it. This workbook enables the student to do fractions. The student should already know how to make the Tic Tac Toe Math grids and use them for multiplication and division.

Tic Tac Toe Patterns

Pattern 1 There is always a five or a zero in the center position.

Pattern 2 All opposite positions on the x and the cross must add to ten.

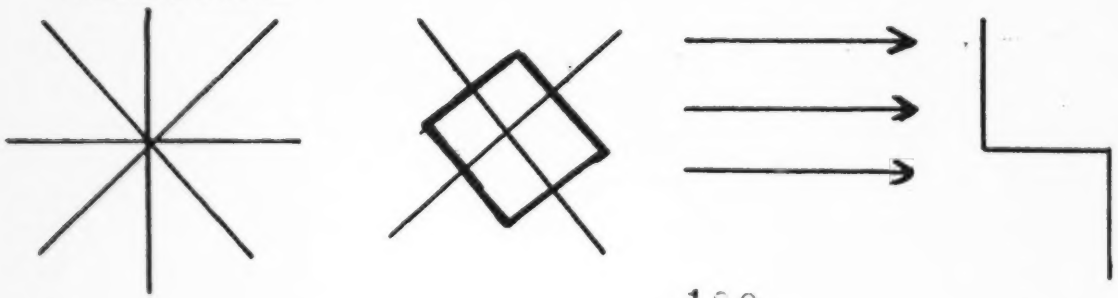
Pattern 3 The second position is obtained by doubling the number in the first position.

Pattern 4 Opposite positions on the X and the cross add up to ten times the first number when the grids are completed.

Pattern 5 Grids of odd numbers always have odd numbers on the X and even numbers on the diamond.

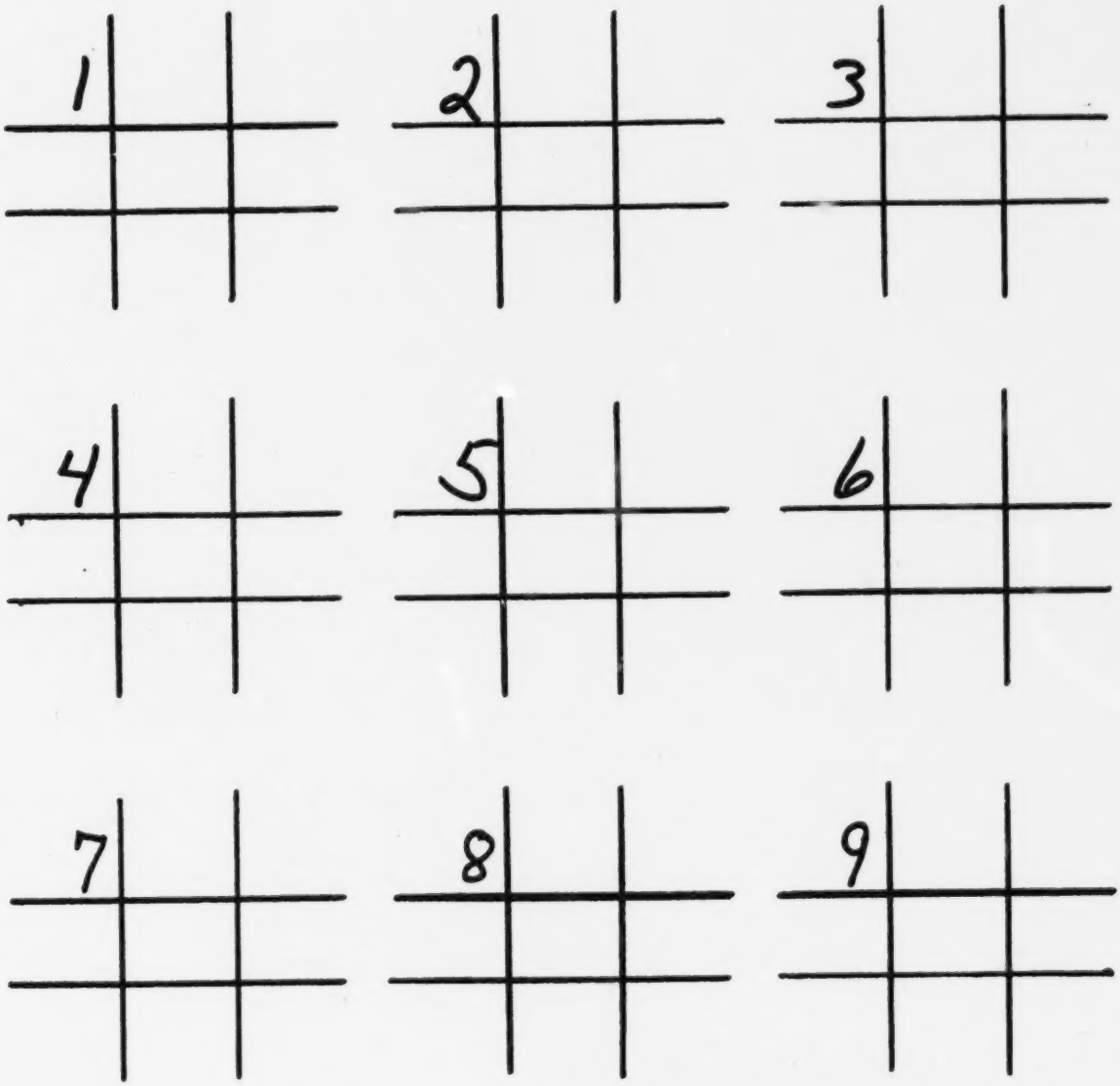
Pattern 6 There are no odd numbers in the even grids.

Visual Pattern



Review of Tic Tac Toe Math

Complete the following grids. If you do not know how to complete these grids, consult the Tic Tac Toe Math Instructional Guide, Tic Tac Toe Math Workbook I or ask your teacher.



Reducing Fractions to the Lowest Terms

Some fractions can be reduced to simpler forms. This means that a fraction written one way may be written in a number of different ways and have the same value. You can learn to write fractions in different ways by simply matching the numbers in the fraction with the positions in a grid. For example:

To change this fraction, look through these grids.

$\frac{15}{27}$

3	12	21
6	15	24
9	18	27

5	20	35
10	25	40
15	30	45

9	36	63
18	45	72
27	54	81

You find 15 in the 3 and 5 grids. You find 27 in the 3 and 9 grids. But you find both numbers in only the 3 grid. Identify the position for each to reduce the fraction. 15 is in the fifth position and 27 is in the ninth position.

$$\frac{15}{27} \qquad \frac{5}{9}$$

Another Example:

$\frac{24}{48}$

Scanning the grids you find that 24 is in the 3, 6, and 8 grids, while 48 is the 6 and 8 grids. Since 48 is not in the 3 grid, you use either the 6 or 8 grid.

3	12	21
6	15	24
9	18	27

6	24	42
12	30	48
18	36	54

8	32	56
16	40	64
24	48	72

Using the 6 grid, identify the position for each number. 24 is in the fourth position, while 48 is in the eighth position. Since 4 and 8 are in the 2 and 4 grid, you can use the 2 or 4 grid. If you use the 2 grid, you find that the numbers are in the second and fourth positions. And again these numbers are in the 2 grid, in the first and second positions or one half.

$$\frac{24}{48} = \frac{4}{8} =$$

2	↓ 8	14
→ 4	10	16
6	12	18

$$\frac{2}{4} = \frac{1}{2}$$

Using the 8 grid, identify the position for each number, This time 24 is in the thrid position and 48 is in the sixth position. These numbers are common to the 3 grid. The 3 is in the first position and the 6 is in the second position. Again providing you with the reduced fraction one half.

6	24	42
12	30	48
18	36	54

8	32	56
16	40	64
24	48	72

2	8	14
4	10	16
6	12	18

$$\frac{24}{48} = \frac{3}{6} \rightarrow \begin{array}{|c|c|c|}\hline 3 & 12 & 21 \\ \hline 6 & 15 & 24 \\ \hline 9 & 18 & 27 \\ \hline \end{array} = \frac{1}{2}$$

A third example:

$\frac{14}{49}$

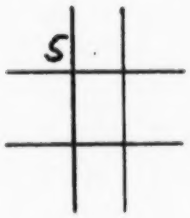
First you find that 14 is in the 2 and 7 grids and 49 is found in the 7 and 9 grids. Both of these numbers are found only in the 7 grid. 14 is in second position and 49 is in the seventh position. Since 2 and 7 do not appear together in any grid, the fraction is reduced to it lowest terms.

2	8	14
4	10	16
6	12	18

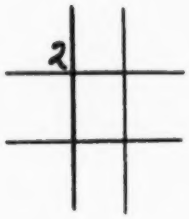
7	28	49
14	35	56
21	42	63

$$\frac{14}{49} = \frac{2}{7}$$

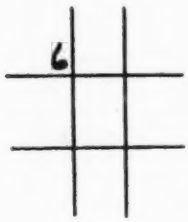
Use the grids to reduce these fractions.

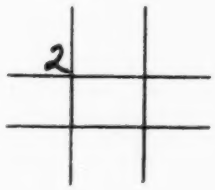
$\frac{15}{25}$ 

$=$ ____

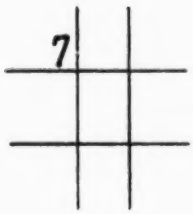
$\frac{2}{8}$ 

$=$ ____

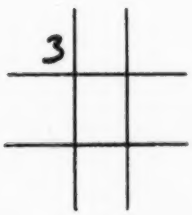
$\frac{12}{24}$ 

$=$ ____ 

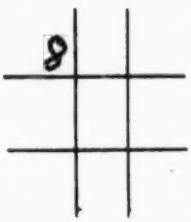
$=$ ____

$\frac{49}{56}$ 

$=$ ____

$\frac{12}{21}$ 

$=$ ____

$\frac{8}{24}$ 

$=$ ____

Changing Mixed Numbers and Improper Fractions

The Tic Tac Toe grids can be used for changing mixed numbers into improper fractions and vice versa. A mixed number is one which has a whole number and a fraction. An improper fraction is one which has a larger number (numerator) on top.

The following are mixed numbers

$2\frac{2}{5}$ $5\frac{1}{4}$ $13\frac{3}{8}$ $1\frac{3}{4}$

These are improper fractions:

$\frac{15}{6}$ $\frac{22}{8}$ $\frac{12}{2}$ $\frac{17}{5}$ $\frac{5}{3}$

Some procedures require improper fractions so you should practice changing them in both directions. To change a mixed number into an improper fraction you need to find or make the grid of the bottom number. For example with the mixed number 3 1/2, you need the 2 grid. Next you look for the position which corresponds to the number in front, (the whole number 3). In the third position there is a 6, you add this number to the number on top (the numerator) 6 + 1 = 7. You rewrite the fraction as 7/2.

2	8	14
4	10	16
6	12	18

$3\frac{1}{2}$ $6+1=\frac{7}{2}$

To change an improper fraction into a mixed number, you again find the grid which matches the bottom number. For example, to change 25/5, you use the 5 grid. Look for the top number in the grid, if it is there, as 25 is in the 5 grid, count the position and you have a whole number without a fraction left over.

$\frac{25}{5}$

5	20	35
10	(25)	40
15	30	45

$= 5$

A way to remember how to do improper fraction is to see the fraction as top heavy. The top number falls into a division problem.

$$5\overline{)25}$$

If the improper fraction was 28/5, you find the two numbers it is between and take the smaller number. 25 is smaller than 30, it is in the fifth position. This provides you will a whole number of 5 with 3 left over. You write the mixed number as 5 3/5.

$\frac{28}{5}$

5	20	35
10	(25)	40
15	30	45

$= 5\frac{3}{5}$

$$5\overline{)28}$$
$$\underline{25}$$
$$3$$

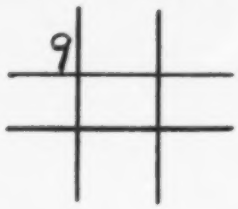
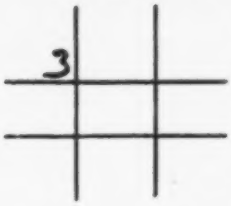
$\frac{13}{3}$

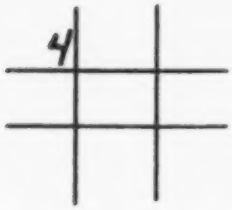
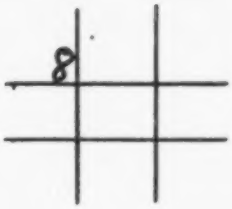
3	(12)	21
6	15	24
9	18	27

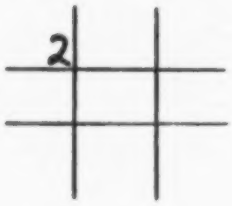
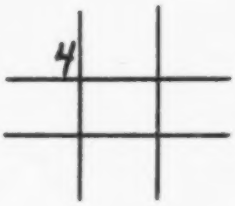
$= 4\frac{1}{3}$

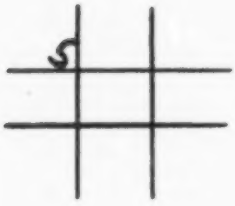
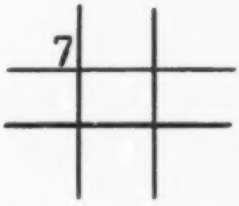
$$3\overline{)13}$$
$$\underline{12}$$
$$1$$

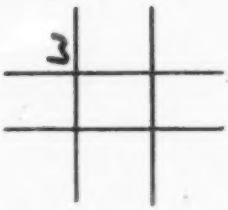
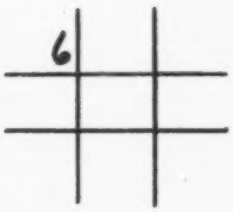
Change the following mixed numbers into improper fractions.

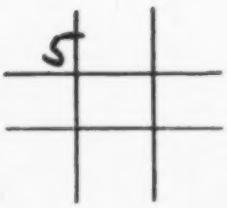
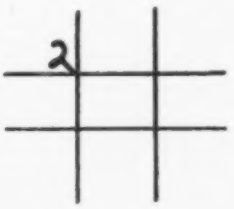
$2\frac{8}{9}$  $3\frac{1}{3}$ 

$4\frac{1}{4}$  $1\frac{3}{8}$ 

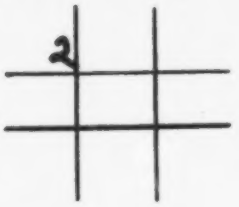
$6\frac{1}{2}$  $2\frac{3}{4}$ 

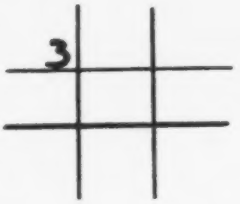
$3\frac{1}{5}$  $1\frac{1}{7}$ 

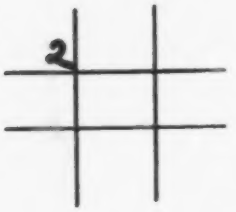
$6\frac{2}{3}$  $2\frac{1}{6}$ 

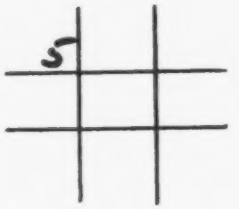
$3\frac{4}{5}$  $2\frac{3}{2}$ 

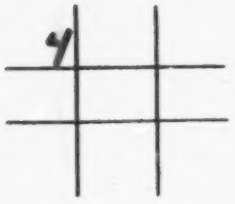
Change the following improper fractions into mixed numbers.

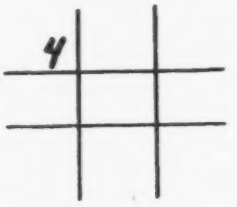
$\frac{10}{2}$ 

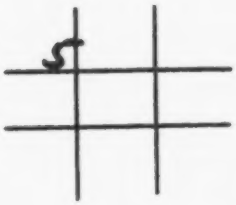
$\frac{11}{3}$ 

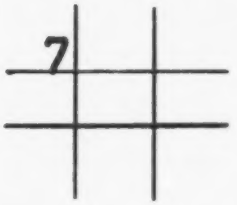
$\frac{7}{2}$ 

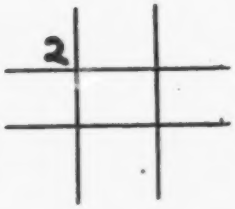
$\frac{15}{5}$ 

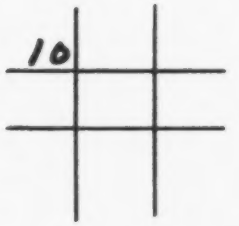
$\frac{8}{4}$ 

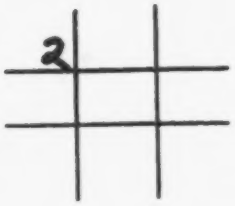
$\frac{9}{4}$ 

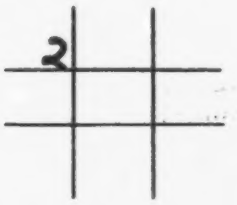
$\frac{12}{5}$ 

$\frac{16}{7}$ 

$\frac{5}{2}$ 

$\frac{13}{10}$ 

$\frac{3}{2}$ 

$\frac{4}{2}$ 

Now it is time to use fractions to solve problems. First multiplication of fractions.

To multipy fractions you simply use the grids to multiply across. Let us look at the following example:

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

4	16	28
(8)	20	32
12	24	36

5	20	35
10	25	40
(15)	30	45

Another Example involves a whole number.

$$\frac{1}{3} \times 3$$

(Note: You need to write the whole number as a fraction. Draw a line under it and place a 1 underneath. Any fraction with the same number on the top and bottom always equals one.)

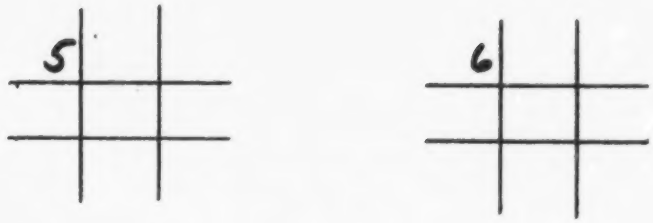
$$\frac{1}{3} \times \frac{3}{1} = \frac{3}{3} = 1$$

(3)	12	21
6	15	24
9	18	27

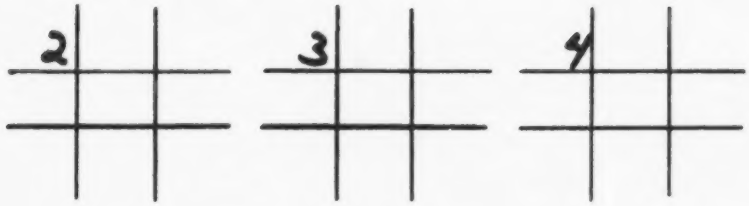
To understand what happens when you multiply fractions,think of having a quarter (25 cents) and multiplying it by three, giving you three quarters or 75 cents.

Here is another example:

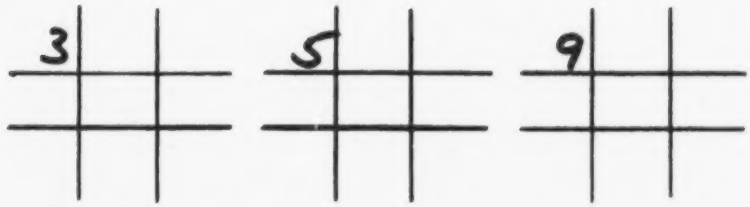
$$\frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$



$$\frac{1}{4} \times \frac{2}{3} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$



$$\frac{5}{9} \times \frac{3}{5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$



Dividing Fractions

Dividing fractions is simple because it is just like multiplying fractions. There is only one additional step, that is to turn the second fraction upside down.

$\frac{1}{2} \div \frac{3}{4}$ changes to $\frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$

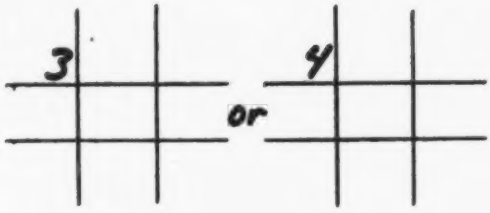
2	8	14
4	10	16
6	12	18

3	12	21
6	15	24
9	18	27

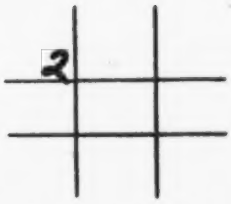
4	16	28
8	20	32
12	24	36

$\frac{1}{4} \div 3$

$\frac{1}{4} \div \frac{3}{1}$ $\quad \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$



$\frac{1}{2} \div \frac{2}{1}$ $\quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$



To see this concretely, imagine that you have 50 cents and you want to divide it in half (by 2) you then have 1 quarter.

Adding Fractions:

When both fractions have the same number on the bottom, add the top numbers and keep the bottom number. Reduce the fraction if necessary.

$\frac{1}{2}$

$\frac{1}{4}$

2	8	14
4	10	16
6	12	18

$$\begin{array}{r} \frac{1}{2} \\ +\frac{1}{2} \\ \hline \frac{2}{2} \end{array} = 1$$

$$\begin{array}{r} \frac{1}{4} \\ +\frac{1}{4} \\ \hline \frac{2}{4} \end{array} = \frac{1}{2}$$

$\frac{2}{3}$

3	12	21
6	15	24
9	18	27

$\frac{9}{8}$

8	32	56
16	40	64
24	48	72

2	8	14
4	10	16
6	12	18

$$\begin{array}{r} \frac{2}{3} \\ +\frac{5}{3} \\ \hline \frac{7}{3} \end{array} \quad 2\frac{1}{3}$$

$$3\overline{)7} \quad \text{remainder } 1$$

$$\begin{array}{r} \frac{9}{8} \\ +\frac{5}{8} \\ \hline \frac{14}{8} \end{array} \quad 1\frac{6}{8} = 1\frac{3}{4}$$

$$8\overline{)14} \quad \text{remainder } 6$$

Subtracting Fractions

When both fractions have the same number on the bottom, subtract the top numbers and keep the bottom number. Reduce the fraction if necessary.

$$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

$$\frac{5}{9} - \frac{2}{9} = \frac{3}{9} = \frac{1}{3}$$

→3	12	21
6	15	24
→9	18	27

$$\frac{6}{4} - \frac{1}{4} = \frac{5}{4} = 1\frac{1}{4}$$

$$\frac{8}{2} - \frac{5}{2} = \frac{3}{2} = 1\frac{1}{2}$$

2	8	14
4	10	16
6	12	18

4	16	28
8	20	32
12	24	36

$4\overline{)5}$ R 1

$2\overline{)3}$ R 1

Common Denominators

If you know the times tables, the common denominator of 1/3 and 1/9 is easy. Finding common denominators is difficult if you do not know the time tables, but, with the grids of Tic Tac Toe Math, it is not only possible but easy.

$\frac{1}{3}$ and $\frac{1}{9}$

Since both 3 and 9 are in the 3 grid, 9 is the common denominator. Since you move 3 positions to get to 9, you multiply both the top and bottom number (1/3) by 3. 1/9 remains the same, 1/3 becomes 3/9.

3	12	21
6	15	24
9	18	27

$\frac{1}{3} \times 3 = \frac{3}{9}$

To find the common denominator of 1/5 and 2/25, we see the common demoninator is in the 5 grid. The 2/25 stays the same, while you move 5 positions to get to 25. 5 x 5 = 25 (clearly seen in the grid).

5	20	35
10	25	40
15	30	45

$\frac{1}{5} \times 5 = \frac{5}{25}$

Practice Page: Find The Common Denominator.

$\frac{1}{3}$ and $\frac{4}{15}$

3	12	21
6	15	24
9	18	27

$\frac{1}{3} \times 5 = \frac{5}{15}$

$\frac{1}{6}$ and $\frac{7}{12}$

6	24	42
12	30	48
18	36	54

$\frac{1}{6} \times$ = ——

$\frac{1}{5}$ and $\frac{2}{35}$

5	20	35
10	25	40
15	30	45

$\frac{1}{5} \times$ = ——

$\frac{5}{9}$ and $\frac{1}{45}$

9	36	63
18	45	72
27	54	81

$\frac{5}{9} \times$ = ——

$\frac{3}{4}$ and $\frac{3}{24}$

4	16	28
8	20	32
12	24	36

$\frac{3}{4} \times$ = ——

$\frac{1}{7}$ and $\frac{5}{56}$

7	28	49
14	35	56
21	42	63

$\frac{1}{7} \times$ = ——

$\frac{2}{10}$ and $\frac{7}{30}$

10	40	70
20	50	80
30	60	90

$\frac{2}{10} \times$ = ——

Adding Fractions with Mixed Denominators:

$\frac{1}{4} \times 2 = \frac{2}{8}$

$+\ \frac{3}{8} \qquad \frac{3}{8}$

$\frac{5}{8}$

4	16	28
(8)	20	32
12	24	36

$\frac{1}{2} \times 2 = \frac{2}{4}$

$+\ \frac{3}{4} = \frac{3}{4}$

$\frac{5}{4} = 1\frac{1}{4}$

2	8	14
(4)	10	16
6	12	18

4	16	28
8	20	32
12	24	36

$\frac{1}{3} \times 3 = \frac{3}{9}$

$+\ \frac{2}{9} \qquad \frac{2}{9}$

$\frac{5}{9}$

3	12	21
6	15	24
9	18	27

$\frac{1}{6} \times 6 = \frac{6}{36}$

$+\ \frac{5}{36} \qquad \frac{5}{36}$

$\frac{11}{36}$

6	24	72
12	30	48
18	36	54

Subtracting Fractions with Mixed Denominators:

$\frac{5}{6}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{7}{8}$

$-\ \frac{1}{2} \times 3$ $\frac{3}{6}$ $-\ \frac{3}{4} \times 2$ $\frac{6}{8}$

$\frac{2}{6} : \frac{1}{3}$ $\frac{1}{8}$

2	8	14
4	10	16
(6)	12	18

4	16	28
(8)	20	32
12	24	36

$\frac{21}{25}$ $\frac{21}{25}$ $\frac{31}{36}$ $\frac{31}{36}$

$-\ \frac{3}{5} \times 5$ $\frac{15}{25}$ $-\ \frac{5}{9} \times 4$ $\frac{20}{36}$

$\frac{6}{25}$ $\frac{11}{36}$

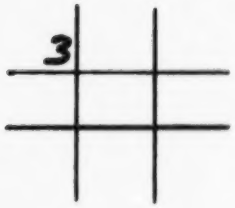
5	20	35
10	25	40
15	30	45

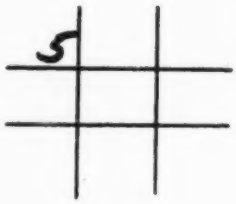
9	(36)	63
18	45	72
27	54	81

Practice Page: Find The Common Denominator and Add.

$\frac{1}{3}$ x ___ ___ $\frac{3}{25}$ ___ ___

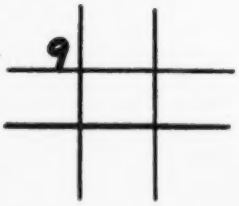
+ $\frac{4}{15}$ ___ ___ + $\frac{2}{5}$ x ___ ___

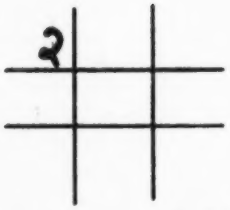


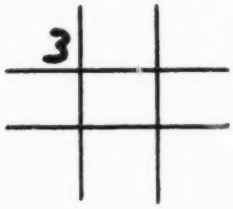


$\frac{1}{9}$ x ___ ___ $\frac{1}{2}$ x ___ ___

+ $\frac{2}{27}$ ___ ___ + $\frac{1}{3}$ x ___ ___







Practice Page: Find The Common Denominator and Subtract.

$\frac{3}{4}$ x ___ ___

$-\frac{1}{8}$ ___ ___

___ ___

4 | | |

$\frac{1}{2}$ x ___ ___

$-\frac{1}{4}$ ___ ___

___ ___

2 | | |

$\frac{7}{35}$ ___ ___

$-\frac{1}{7}$ x ___ ___

___ ___

7 | | |

$\frac{9}{16}$ ___ ___

$-\frac{3}{8}$ x ___ ___

___ ___

8 | | |

Finding Common Denominators Using Two Grids

When the denominators (the bottom number of a fraction) are not in the same grid, you need to use two grids. Let's look at the following example.

$$\frac{1}{3} \qquad \frac{1}{4}$$

Since 4 and 3 are not in the same grid, you need to find a number which is common to both grids. In this case, 12 and 24 are in both grids. Both are common denominators, but usually you will use the smallest one.

3	(12)	21
6	15	(24)
9	18	27

4	16	28
8	20	32
(12)	(24)	36

To change these two fractions to have common denominators, multiply each fraction by the number of the position from the grid of the denominator. One thrid uses the three grid and one fourth uses the four grid.

$$\frac{1}{3} \times 4 = \frac{4}{12}$$

$$\frac{1}{4} \times 3 = \frac{3}{12}$$

Examples:

$\frac{1}{3}$ and $\frac{1}{5}$ $\quad$ $\frac{1}{6}$ and $\frac{1}{9}$ $\quad$ $\frac{2}{3}$ and $\frac{1}{7}$

$\frac{1}{3} \times 5 = \frac{5}{15}$ $\quad$ $\frac{1}{6} \times 6 = \frac{6}{36}$ $\quad$ $\frac{2}{3} \times 7 = \frac{14}{21}$

$\frac{1}{5} \times 3 = \frac{3}{15}$ $\quad$ $\frac{1}{9} \times 4 = \frac{4}{36}$ $\quad$ $\frac{1}{7} \times 3 = \frac{3}{21}$

1	4	7
2	5	8
3	6	9

4	16	28
8	20	32
12	24	36

7	28	49
14	35	56
21	42	63

2	8	14
4	10	16
6	12	18

5	20	35
10	25	40
15	30	45

8	32	56
16	40	64
24	48	72

3	12	21
6	15	24
9	18	27

6	24	42
12	30	48
18	36	54

9	36	63
18	45	72
27	54	81